A hybrid decision-making model for optimal portfolio selection under interval uncertainty

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Abstract

This paper aims to propose a hybrid approach based on a fuzzy multi-criteria decision making and a multi-objective mathematical optimization under interval uncertainty to solve the investment management problem in the Iranian capital market. For this purpose, first, a fuzzy-SWARA method is utilized to determine the global importance weights of criteria. Then, a fuzzy-EDAS method is developed to rank the active industries in the Iranian capital market including basic metals, chemical products, investment, metal ore mining, financing, insurance and pension funds, and except social security. Second, a mathematical model is presented to determine the optimal amount of investment in each ranked alternative. According to the numerical results, access to financial resources, access to distribution networks and access to raw materials are the most important criteria in evaluating different areas of investment. The highest optimal share of investment is related to Fars 1 and the lowest value is related to Gharn1. In solving the model in conditions of uncertainty, it is observed that changing Γ_1 from small to large values reduces the value of the first objective function in the most efficient Pareto member. While in $\Gamma_1 > 10$, the first objective function value is fixed. The third objective function also has an ascending trend with the descending changes of parameter Γ_3. The obtained results can be considered as a managerial tool for participation of the research.

Keywords: SWARA-fuzzy EDAS, Robust Optimization, Multi-objective Optimization, Iranian Capital Market

1- Introduction

Generally, production and trade play an important role in economic environment and can be considered as engines of the economy to the country's survival in domestic and foreign markets. Accordingly, the proper strengthening and utilization of productive and commercial capacities and the creation of new capacities, while paving the way for development, production and provision of services, also provide the basis for sustainable economic development (Thoumi, 2009). Therefore, the role of the government as a supporter of guidance programs in production and trade needs to be more colorful. By creating a common ideal between those in charge, it is possible to provide support for production, employment, and productive and commercial investments to achieve self-sufficiency. On the other hand, by supporting products with export development potential with the cooperation and participation of the private sector, it is possible to take advantage of the existing capacities and to improve productivity, while achieving self-sufficiency in order to enter and penetrate global markets (Khodaverdizadeh & Mohammadi, 2016).

It should be noted that the investment problem in domestic production has always been considered as one of the most important criteria for economic development societies (Thoumi, 2009). There are some significant advantages including creating sustainable employment, developing industry and increasing GDP, reducing dependency on imported industries, developing exports and currency appreciation, and creating a suitable platform for the development of other service sectors (Allcott & Keniston, 2018). The history of industrial development in Japan and Germany in the 19th century can be referred to as successful world experiences. During World War II, Germany and Japan, due to global sanctions, were unable to meet their industrial needs and were forced to produce needed products based on domestic capabilities. Due to the achievements of these countries, this issue has gradually grown as a culture of national development and has been considered by many researchers in the industrial development literature (Liza & Morales Anaya, 2018). Today, the concept of domestic industry development is known as a model of progress in many countries, including Iran, but its implementation requires long-term planning based on scientific knowledge. This issue has wide dimensions and cannot be achieved with a short-term view of the intended goals. Therefore, there is a need for planning in various industrial and commercial sectors. As a desirable goal, it should be imagined that all the products that are needed by the society and the potential for their production is available domestically, and they should be provided with the help of internal forces. This issue can be achieved more appropriately with the help of transferring technical knowledge from other countries (Popkova, Bogoviz, Ragulina, & Alekseev, 2018). In fact, it should be noted that the development of a country is not possible in isolation and requires interaction with other countries. Therefore, in the development of the national economy, the situation of the world market, international relations and trade relations should also be considered.

According to the literature, the research gap of the paper is related to investigate sustainable development in stock markets with the help of quantitative models. In fact, organizations responsible for promoting sustainable production have a duty to create suitable opportunities that safeguard financial and human resources domestically, resulting in the movement of economic cycles. Despite the crucial role of sustainable development in the stock market, there has been no investigation of this issue in the literature. In Iran, the lack of budget and economic sanctions is causing a decline in investment incentives and an increase in unproductive employment, which will lead to future difficulties. One of these problems is the country's heavy reliance on imported goods due to a lack of enthusiasm for domestic production, which needs to be addressed by conducting both theoretical and practical research to safeguard existing capital in the production sector.

In this study, the investment management problem in the Iranian capital market is investigated by utilizing a hybrid approach based on fuzzy-MCDM and optimization model under interval uncertainty. In the first phase, with the help of a fuzzy-SWARA and a fuzzy-EDAS the importance of the criteria and evaluation of different areas of investment are determined. Fuzzy-EDAS method prioritized each of the selected alternatives. Then, using a multi-objective optimization model under interval uncertainty, the optimal amounts of investment in each company are determined. Finally, in order to perform managerial analysis and provide decision-making policies, various numerical analyzes are performed.

In the remain parts of the paper, first the research literature is investigated in detail in section 2. Then the proposed methods including fuzzy-MCDM and optimization model is stated in the section 3. Numerical results are described in section 4 and quantitative analysis are performed to present managerial insights. Finally, the section 5 a conclusion and some future suggestions is described.

2- Literature review

The continuous growth of the world's population, lack of resources and environmental pressures are important factors in determining the transition to greener and more sustainable planets (Mansley, 2000). Over the past decade, governments around the world have addressed climate change issues by revitalizing the national economy through sources of sustainable economic, social, and environmental growth (Kisman & Krisandi, 2019). In the 2015 Paris Agreement, countries agreed to strengthen the global response to climate change threats by maintaining global temperatures (Arif et al., 2020). To move towards low-carbon economies and to reduce poverty and sustainable livelihoods, investment in green employment, biodiversity conservation, renewable energy, sustainable water management and waste management must be implemented nationally. However, advanced economies have recently suffered from the lack of investment in public infrastructure, while developing economies do not have access to modern services for their growing populations (Caplan, Griswold, & Jarvis, 2013). Accordingly, the ability to raise the right type of investment for the infrastructure sector is crucial. Climate policymakers are therefore responsible for creating incentives to promote green growth and encouraging private sector investment in sustainable projects (Shabbir & Wisdom, 2020). The growing importance of sustainable and environmental investments in financial markets also has implications. Financial markets are responding to the growing demand for low-carbon projects around the world to meet the challenges of climate change. In fact, new financial instruments have been developed with the aim of directing capital to green projects. Mathematical optimization can be used to finance low carbon and healthy climate resistant infrastructures (Arif et al., 2020). The following are some of the most recent studies in the field of sustainable investment management.

Cesarone et al. (2019) examined the issue of stock portfolio selection by considering risk management criteria. In order to solve the problem, they presented a hybrid approach based on simulation and optimization methods. In this approach, a greedily classical single-discipline innovative algorithm is used that can produce appropriate solutions. According to the numerical results, it has been observed that the criteria related to risk management had a much greater impact on the final output than the economic criteria. Castilho et al. (2019) proposed a method based on the classical mean-variance analysis using machine learning in order to optimize the stock portfolio selection problem in stock exchange networks. Uncertain future returns and PER ratios of each asset are approximated using fuzzy L-R numbers, as well as budget, scope, and cardinality constraints. Galankashi et al. (2020) used the fuzzy analytical network process method to evaluate and select a stock portfolio on Tehran Stock Exchange. First, a literature review was performed to determine the main criteria for selecting the portfolio, and then a Likert questionnaire was used to finalize the list of criteria. Final criteria were applied in the fuzzy analytical network process to rank 10 portfolios. The results showed that profitability, growth, market and risk are the most important criteria for choosing a portfolio. Vuković et al. (2020) compared stock portfolio selection using a combination of multi-criteria decision making and modern portfolio theory, which includes only Croatian capital market indicators. The results show that there was a significant difference in stock rankings. However, stocks that were not included in any portfolio

in the selection of the modern portfolio theory were ranked lowest due to the MCDM hybrid approach, which confirmed that these stocks were for investment in the worst-case scenario. Rezaei Nokandeh et al. (2020) presented a hybrid model consisting of three steps: 1) coverage analysis (for initial stock revision), 2) multi-criteria decision making (TOPSIS) in conditions of uncertainty and 3) presentation of planning model. In order to select the best stock portfolio according to the priorities and constraints of the organization, they provided a line to achieve the highest compatibility between the final selection and the initial ranking of each share. Xu et al. (2020) selected a portfolio of renewable energy desalination systems with a sustainable perspective within a multi-criteria decision-making framework under data uncertainty. A mathematical framework was proposed to deal with data uncertainty. A fuzzy network analysis method was used to assign weight to related criteria. Finally, the logical ranking of the options was done. Stanković et al. (2020) stated that despite the widespread use of modern stock portfolio theory and Markowitz's approach for optimization, which is based on quadratic planning and the distribution of probability returns as key parameters, these approaches have been criticized. The standard mean variance, has been modified using more appropriate risk criteria in the optimization algorithm, which has been tested in portfolio management on the Belgrade Stock Exchange. Doaei *et al.* (2021) predicted daily Tehran Exchange Dividend Price Index (TEDPIX) via the hybrid multilayer perceptron (MLP) neural networks and metaheuristic
Algorithms. The results showed that grey wolf optimization has superior performance to train MLPs for predicting the stock market in metaheuristic-based. Yoshino et al. (2021) examined the impact of the Covid virus 19 and the achievement of sustainability goals on the stock portfolio issue. This article theoretically shows that the current allocation of investors by considering sustainability goals based on different consulting firms would lead to a change in the investment portfolio. The allocation of stocks can be done globally by taxing pollution and waste such as CO2, NOx and plastics at the same tax rate, and the global pollution tax would lead to the allocation of stocks. Doaei and Saberfard (2021) investigated stock portfolio selection in Iran capital market by uncertainty conditions. They found out in both multi-objective and single-objective situations can be implemented in real-world conditions and it can be said that the use of computational results of this study can be used as an operational tool. Gua et al. (2021) examined selecting a high-order Markov stock portfolio with a capital gains tax. In this paper, capital gains-losses compensation has been studied with respect to the effect of loss transportation. Markov switching mechanism is offered in the market of different countries. The average variance model of Markov switching order is made randomly. A particle swarm optimization algorithm based on Monte Carlo simulation was proposed to solve the problem. Mostafae and Doaei (2022) optimized the portfolio in listed companies on Tehran Stock Exchange and Iran Farabours as a multi-objective optimization problem. The numerical results showed it can be seen that the gray wolf algorithm has a higher efficiency than the genetic algorithm in all examples.

According to the above description, one of the obligations of the organizations in charge of sustainable production development is to create appropriate opportunities to protect human and financial resources at home, which leads to the movement of economic cycles. Currently in Iran, due to the lack of budget and existing sanctions, the incentive to invest is decreasing day by day and the tendency to invest in unproductive employment is strengthening. This will cause many problems in the future. Among them, we can mention the strong dependence of the country's consumer market on imported products due to the loss of the spirit of production boom. Therefore, it is necessary to conduct theoretical and practical studies to protect existing capital in the production sector. Therefore, in this research, a hybrid model based on multi-criteria decision making and multi-objective optimization for accurate investment in different production sectors is proposed. The main aim of this study is to improve the current situation of investment, using mathematical decision-making and optimization tools. The main contributions of this research are as follow.

1. Providing a hybrid model of multi-criteria decision making and multi-objective optimization

2. Determining high priority companies for the gradual transfer of capital from the private sector

3. Using mathematical planning methods to determine the volume of investment by considering multiple goals

4. Using fuzzy programming in decision making and robust optimization in mathematical modeling

5. Considering the conditions of uncertainty in some input parameters of the problem

3- Research method

The economic and financial situation of global and international markets which are often due to the outbreak of the Corona virus in 2020, have left private sector investors with many problems to direct capital to financial markets (Ferneini, 2020). In fact, the decision-making criteria that investors have considered in previous years for the optimal selection of stock portfolios cannot now lead to highly reliable answers (Talan & Sharma, 2019). In general, the criteria to measure the performance of manufacturing and investment companies can include attention to economic trends, employment infrastructure and criteria related to the social dimension. However, the question that needs to be answered in the first stage is how to limit the scope of decision-making on choosing the right companies to invest in so that the optimal composition of the stock portfolio can be created with more focus. In fact, it is very important to be able to conduct initial screening to eliminate weaker companies before a thorough analysis of companies operating in the financial markets to direct capital to them. This is precisely a decision based on a set of management criteria and sub-criteria, the output of which leads to limiting the number of potential companies to invest in (Ho, Tsai, Tzeng, & Fang, 2011). The level of need to examine the issue of this research can be found in the turmoil in the Iranian financial markets. At present, the use of the former analysis methods does not meet the needs of investors to provide reliable answers. In other words, some numerical analyzes may show the conditions for a company to grow in the future, but what actually happens is the opposite, and the directed investment in that company is virtually lost. One of the main reasons for this problem is the consideration of some criteria for evaluating investment in various areas active in the capital market. Therefore, providing a suitable approach to consider a wider range of information and criteria in order to obtain final answers can to a large extent lead to high-reliability answers. Some of the benefits of conducting this research can be considered in providing highly reliable answers to determine the share of investment in different companies. In fact, the implementation of this research will create a broader view of decision-making criteria in

this area, as well as the use of new tools. In addition, the high flexibility of the proposed approach can pave the way for its improvement and the introduction of more criteria and sub-criteria.
The proposed framework of this research consists of two phases. In the first phase, using a multi-criteria decision-making model, various industries of the Iranian capital market, including basic metals, chemical products, investment, metal ore extraction, financing papers and social security insurance and pension fund are evaluated. Then a mathematical optimization model is developed to determine the amount of investment in each company according to different objective functions. Therefore, the final outputs can be provided research beneficiaries as investment management decisions. In order to ensure the obtained solutions, the necessary sensitivities are analyzed for examining the behavior of the proposed framework in different situations. Figure 1 shows the flowchart of the research method used in the paper.

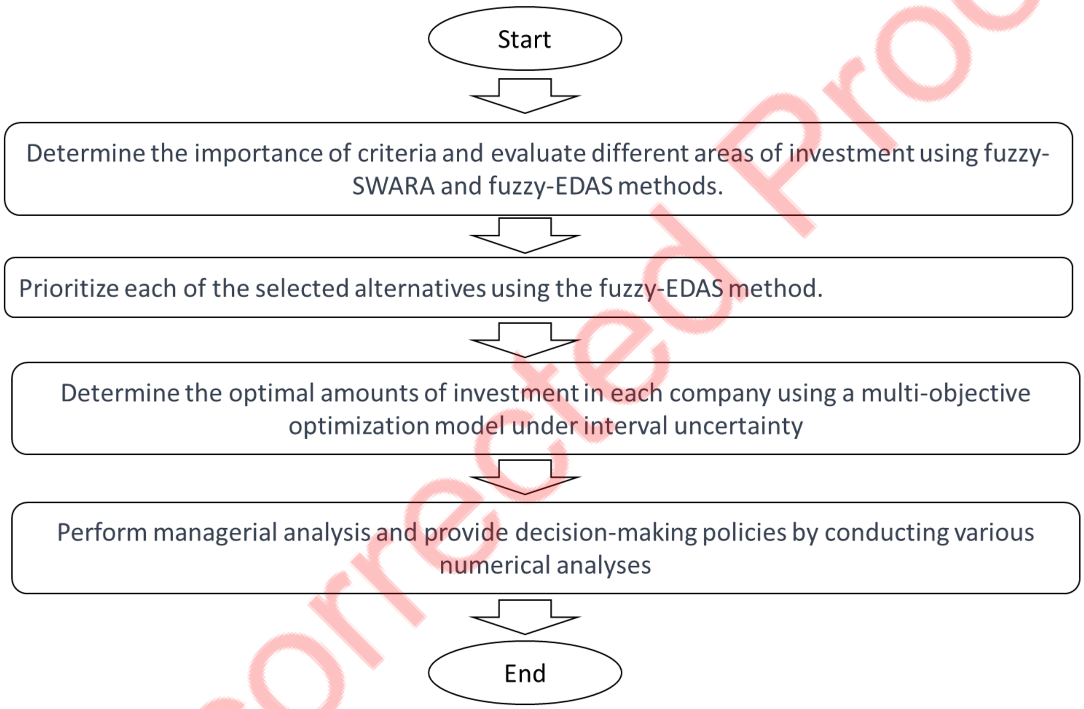

Figure 1. flowchart of the research paper

3-1- The first phase: multi-criteria decision making

In the majority of MCDM processes, the decision-makers provide indefinite responses rather than exact and precise solutions (Farughi & Mostafayi, 2017) and (Li & Zhao, 2016). In fact, every decision-making problem comes with particular uncertainties and ambiguities that arise from the subjective judgments performed by the decision-makers. Such uncertainties are even more likely in problems where the criteria are dominantly expressed in qualitative terms. On the other hand, many times the decision-making models based on the decision-makers' subjective judgments render inaccurate since they need great deals of relevant knowledge, experience, and expertise and (Banaeian, Mobli, Fahimnia, Nielsen, & Omid, 2018). Therefore, in order to treat such problems appropriately, it makes more sense to utilize the fuzzy set theory and linguistic terms, rather than traditional methods, to score various preferences. In this section, we begin by explaining the fundamental definitions for the fuzzy set theory before introducing the fuzzy methods of SWARA and EDAS in separate subsections. Finally, the provided definitions are compiled to develop a hybrid SWARA-EDAS MCDM model in a fuzzy domain.

3-1-1- Fuzzy SWARA method

Different multi-criteria decision-making methods have been used by researchers to determine the weight of criteria in recent years, such as Analytic Hierarchy Process (AHP), Analytical Network Process (ANP), Decision-Making Trial and Evaluation Laboratory (DEMATEL), Simple Multi-Attribute Rating Technique (SMART), Weighted Sum Method (WSM), the best-worst method (BWM) and others (Ansari, Kant, & Shankar, 2020). The Step-wise weight assessment ratio analysis (SWARA) is one of the multi-criteria decision-making methods based on determining the weight of criteria (Keršuliene, Zavadskas, & Turskis, 2010). The main advantage of SWARA is its ability to evaluate the opinions of experts and estimation of the relative importance of each criterion. The importance of criteria is also often judged by the weight priorities derived from the pairwise comparison matrix (Kou, Ergu, Lin, & Chen, 2016; Kou, Peng, & Wang, 2014). In the SWARA method, experts can freely evaluate criteria without using a scale. One of the features of SWARA method is the number of pairwise comparisons with AHP, ANP or even BWM methods. In fact, in this method, the number of pairwise comparisons when n criteria are ranked in descending order according to their importance is equal to $n-1$ (Keršuliene et al., 2010). While in AHP method, $n\ (n-1)$)Mardani et al., 2017() and in BWM, $2n-3$ pairwise comparisons are performed (Rezaei, 2015, 2016). Also, the SWARA method ranks the criteria in descending order, so there is no need to examine the consistency of the judgments. SWARA can be easily organized in complex or abnormal situations to control inaccurate and ambiguous information using a fuzzy approach. The procedure for achieving the relative weights of the criteria using the fuzzy SWARA method is presented in section (b) of the article appendix.

3-1-2- Fuzzy EDAS method

The Evaluation based on Distance from Average Solution (EDAS) method is a multi-criteria decision-making method introduced by (Keshavarz Ghorabaee, Zavadskas, Olfat, & Turskis, 2015). This method was first used to classify inventory items by several criteria. However, they showed that the EDAS method is also effective to deal with multi-criteria decision-making problems in a general context (Ghorabaee, Amiri, Zavadskas, & Antucheviciene, 2018). The evaluation of alternatives in this method is based on the distance of each alternative from the average solution to each criterion. The mean solution in this method is a practical solution that includes the average of the elements obtained in each criterion. The desirability of solutions (alternatives) in the EDAS method is calculated based on the positive and negative distances of the mean solution. Each alternative has a positive and a negative distance with the mean solution for each criterion and these distances are calculated according to the nature of the criteria. The alternative with more positive distance and less negative distance from the mean solution is the best one. Due to the ambiguity in decision making, the application of fuzzy concept in MCDM can lead to more reliable decision results. The developed fuzzy EDAS method is a new and efficient method to deal with multi-criteria decision problems in an uncertain environment with fuzzy information (Ghorabaee, Zavadskas, Amiri, & Turskis, 2016). In order to evaluate the alternative for each criterion, the fuzzy rating range presented in Table 1 has been used. The process of solving the fuzzy EDAS method includes the steps presented in section (c) of the article appendix, which is based on research (Polat & Bayhan, 2020; Stević, Vasiljević, Zavadskas, Sremac, & Turskis, 2018).

Table 1- Linguistic expressions to determine the priority of alternatives

Linguistic terms	Very low (VL)	Low (L)	Medium (M)	High (H)	Very High (VH)
TFNs	(1, 1, 1)	(2, 3, 4)	(4, 5, 6)	(6, 7, 8)	(8, 9, 9)

3-2- The proposed mathematical model

Choosing investment alternative is a complex decision that requires the use of optimal solutions to achieve the goals of investors (Couture & Gagnon, 2010). Therefore, the development of mathematical models can be used as the best decision-making tool (Darmian & Farughi, 2022). It should be noted that entering data in raw form reduces the speed and accuracy of the solution method. To avoid this situation and also in order to equalize the value of the data, the input data must be normalized before the test. All data must be normalized between 1 and -1. In this research, the data are normalized before testing the model and then the solution algorithm is examined by MATLAB software.

$$Y_i = \frac{y_i - y_{\min}}{y_{\max} - y_{\min}}(h_i - L_i) + \mathrm{L}_i$$

Y_i	Normalized input values in the middle of the equation
y_i	Main input values
y_{min}	The smallest amount of input
y_{max}	The largest amount of input
h_i	High value at normalization interval (+1)
L_i	Low value at normalization interval (-1)

Finally, the formulation of this problem is described as follows.

Sets and indices

$i \in \{1, \dots, I\}$	set of potential companies

Input parameters

P_i	Priority of each company *i*
L_i	Minimum percentage of desired investment in company *i*
U_i	Maximum percentage of desired investment in company *i*
$Budget$	The total budget available for the allocation of financial incentives
$Income_i$	Annual income from investing in company *i*
$cost_i$	Annual investment cost in company *i*
β_i	Investment risk in company *i*
N	Maximum number of companies to invest
M	Positive numerical and large enough

Decision variables

y_i	Amount of financial incentives allocated by the government to company *i*
x_i	Amount of investment in company *i*

w_i equal to 1 if company i is selected for investment and otherwise equal to zero.

$$Max\ Z \tag{1}$$

$$Max\ Z_2 = \sum_{i \in I} \frac{(Income_i + y_i) - cost_i}{cost_i} \times x_i \tag{2}$$

$$Min\ Z_3 = \sum_{i \in I} \beta_i \times x_i \tag{3}$$

$$s.t.$$

$$Z \leq P_i \times x_i \tag{4}$$

$$L_i \leq x_i \leq U_i \qquad i \in I \tag{5}$$

$$\sum_{i \in I} y_i = Budget \tag{6}$$

$$y_i \leq M \times w_i \qquad i \in I \tag{7}$$

$$w_i \leq M \times y_i \qquad i \in I \tag{8}$$

$$x_i \leq M \times w_i \qquad i \in I \tag{9}$$

$$w_i \leq x_i \qquad i \in I \tag{10}$$

$$\sum_{i \in I} x_i = 1 \tag{11}$$

$$\sum_{i \in I} w_i \leq N \tag{12}$$

$$w_i \in \{0,1\}\ and\ 0 \leq x_i \leq 1 \qquad i \in I \tag{13}$$

$$y_i \geq 0 \qquad i \in I \tag{14}$$

The first objective function maximizes the minimum investment made in companies. In fact, according to constraint (4), the variable Z represents the minimum investment commensurate with the market value, which is maximized in the objective function. The second objective function is to maximize the revenue-to-cost ratio in companies. Function Given that many banking financial systems are based on annual intervals, this function calculates the target return on an annual basis. The third objective function minimizes the investment risk in companies. The amount of investment risk can be calculated based on the geometric mean of the deviation from the criterion of the amount of stock returns of active companies. Constraint (5) ensures that the level of investment must be within the government's range. Constraint (6) ensures that the total amount of financial incentives allocated to each company is equal to the total available budget. Constraints (7) and (8) ensure that financial incentives can be assigned to a company when that company has been selected for investment. Constraints (9) and (10) guarantee that if a company is selected for investment, a percentage of private sector capital must be invested in it. Constraint (11) ensures that the total investment in companies is equal to 1. Constraint (12) ensures that the maximum number of companies selected for investment is limited to N. Constraints (13) and (14) indicate the range of decision variables.

3-2-1- The mathematical model under uncertainty

Based on the literature, various methods have been proposed to control the level of uncertainty to estimate the exact value of some parameters. Robust programming is known as one of the most effective approaches (Darmian, Fattahi, & Keyvanshokooh, 2021).

Interval Robust Optimization (IRO) is a type of optimization technique that is designed to handle uncertainty or imprecision in the input data of a model. In traditional optimization, the input

parameters are assumed to be precise and exact, which is not always the case in real-world applications. Interval uncertainty arises when the values of input parameters are known only to lie within some known interval or range, rather than being known exactly. IRO is a methodology that allows for optimization under interval uncertainty by considering a set of possible values for each input parameter. These sets of possible values are called "uncertainty sets," and IRO seeks to optimize the worst-case outcome over all possible values of the input parameters. In other words, the objective is to find a solution that is feasible for all possible values of the input parameters within their respective uncertainty sets. IRO can be particularly useful in situations where there is significant uncertainty about the input parameters, such as in financial modeling, supply chain management, or environmental management. By accounting for interval uncertainty, IRO can provide decision-makers with more robust and reliable solutions that are less sensitive to variations in input parameters (Farughi & Mostafayi, 2016).
One of the main challenges in IRO is to find an appropriate uncertainty set for each input parameter. The choice of uncertainty set can significantly impact the results of the optimization, and finding an appropriate set often requires domain-specific knowledge and expertise (Farughi, Dolatabadiaa, Moradi, Karbasi, & Mostafayi, 2017). In this study, a robust optimization tool based on Bertsimas model is developed to face with the uncertainty in parameter of risk.
Since the parameter β_i always has inherent uncertainty, in this study, in order to deal with the uncertainty in these parameters, the robust programming method is used (Bertsimas & Sim, 2004). The structure of this method is such that each parameter is set in an interval with specified upper and lower bounds, although there is no information on how to distribute the data in this interval. The parameters of the problem change as follows.

$$\widetilde{\beta_i} = [\beta_i - \widehat{\beta_i}, \beta_i + \widehat{\beta_i}] \qquad 15$$

where $\widetilde{\beta_i}$ is the value of the parameter under uncertainty, β_i is the mean value of the parameter in the defined interval, and $\widetilde{\beta_i}$ is the mean deviation of the mean for the parameter. From a mathematical programming point of view, it is possible to transform an uncertain problem into a certain one through a nonlinear polynomial function as shown below (Bertsimas & Sim, 2004).

$$\max_{X_i \in f(X)} \left(\sum_{i=1}^{I} \beta_i X_i - \overbrace{\max_{\left\{\substack{S: S \subseteq I, |S| \le \Gamma \\ (i_t \in I \setminus S)}\right\}} \left(\sum_{(i) \in S} \widehat{\beta_i} X_i + \left(\Gamma_1 - \lfloor \Gamma_1 \rfloor\right) \widehat{\beta_{i_t}} X_{i_t} \right)}^{\theta} \right) \qquad 16$$

Given that the above equation is nonlinear, this equation cannot be solved accurately, and therefore it needs to be converted to a linear one. In the method presented by (Bertsimas & Sim, 2004), a constant parameter Γ is defined which is set in the interval $[0, |I|]$. This parameter is a kind of controller of uncertainty limits in equations where uncertainty parameters are present. If $\Gamma = 0$, it means that there is no uncertainty in the problem and in fact the same state of input parameters is obtained. But if $\Gamma = |I|$, it means that the problem has the highest level of uncertainty and it is similar to Soyster's problem-based programming problem)Soyster, 1973(. Therefore, it is necessary to analyze the different levels of uncertainty in the values $0 < \Gamma < |I|$. In order to linearize the above equation, a mathematical theory is presented and the steps of its proof are described.

Theory: the presented mathematical model, considering equation (16) as objective functions, is compatible with the formulations provided for the Robust Model.

Robust Model

$$Min\ Z = CVaR(x) - \Gamma_1\, U_0^1 + \sum_{i=1}^{I} UR_i^1 \qquad 17$$

$s.t.$

Constraints 6 to 8 $\qquad$ 18

$$U_0^1 + UR_{ij}^1 - \hat{r}_i\, X_i \geq 0, \qquad i \in I, j \in J \qquad 19$$

$$UR_{ij}^1 \geq 0, \qquad i \in I, j \in J \qquad 20$$

$$U_0^1 \geq 0, \qquad 21$$

Proof: For a given value of $(X_i)_{i=1,\dots,I}$, the θ part of Equation (16) can be linearized using the definition of the variable Z_i^1 with a range of $0 \leq Z_i^1 \leq 1$. Thus, the nonlinear structure of Equation (16) can be considered equivalent to Model 1.

Model 1

$$min \sum_{i=1}^{I} \hat{r}_i\, X_i Z_i^1 \qquad 22$$

$s.t$

$$Z_i^1 \leq \Gamma_1 \qquad i \in I \qquad 23$$

$$0 \leq Z_i^1 \leq 1 \qquad i \in I \qquad 24$$

The optimal solution for each of these formulations must have $\lfloor \Gamma \rfloor$ variable $Z_i^{obj} = 1$ and a $Z_i^{obj} = \Gamma - \lfloor \Gamma \rfloor$ which is equivalent to the optimal solution in part θ. Using a strong duality for the given values $(X_i)_{i=1,\dots,I}$, Model 1 can be rewritten linearly equivalent to Model 4.

Model 4

$$Min\ Robust_1 = \Gamma_1\, U_0^1 + \sum_{i=1}^{I} UR_i^1 \qquad 27$$

$s.t$

$$U_0^1 + UR_i^1 - \hat{r}_i\, X_i \geq 0, \qquad i \in I \qquad 28$$

$$UR_i^1 \geq 0, \qquad i \in I \qquad 29$$

$$U_0^1 \geq 0, \qquad 30$$

Combining Model 4 with Equation (16), respectively, results in the Robust Model and thus the proof is done.

4- Numerical results of multi-criteria decision phase

This section describes the numerical results obtained from the implementation of the proposed multi-criteria decision model. For this purpose, first, the results of the Fuzzy SWARA method are expressed to determine the score of each criterion and sub-criterion. The final prioritization of alternatives is then determined using the EDAS method. In Table 2, the set of research criteria and sub-criteria is determined.

Table 2 - Criteria and sub-criteria related to the evaluation of industries active in the capital market

Criteria	Sub criteria	Reference
Resources and ability of the organization to create a competitive advantage (C1)	Price competitiveness (C11)	(Mkwanazi, 2018), (Balali, Saadi, & Ghazvineh, 2015)
	Dedicated access to finance (C12)	(Balali et al., 2015), (Ali, Agyekum, & Adadi, 2021)
	Access to suitable distribution networks (C13)	(Mkwanazi, 2018)
	Efficient R&D (C14)	(Ali et al., 2021)
	Financial strengths (C15)	(Ram & Montibeller, 2013), (Gudo, Deng, Belete, & Abubakar, 2020)
External Environment Opportunities (C2)	Potential customers (C21)	(Mkwanazi, 2018) , (Gudo et al., 2020; Rais, Acharya, & Sharma, 2013)
	Use of new technologies (C22)	(Mkwanazi, 2018)
	Reducing legal restrictions (C23)	(Gudo et al., 2020)
	Removing barriers to world trade (C24)	(Ram & Montibeller, 2013), (Rais et al., 2013)
	Potential competitors (C25)	(Mkwanazi, 2018)
Key and strategic inadequacies (C3)	Being unknown among customers (C31)	(Gudo et al., 2020)
	Raw material access problem (C32)	(Mkwanazi, 2018), (Ali et al., 2021)
	Instability in production (C33)	(Balali et al., 2015)
	Weak industrial relations (C34)	(Balali et al., 2015), (Ali et al., 2021)
	Consecutive management problems (C35)	Expert's opinion
Environmental hazards and constraints on industries (C4)	Ability to change products to suit customer tastes (C41)	(Ram & Montibeller, 2013), (Rais et al., 2013)
	Ability to produce high-power alternative products (C42)	Expert's opinion
	Increasing trade restrictions (C43)	(Ram & Montibeller, 2013), (Rais et al., 2013)
	Government and Administrative Bureaucracy (C44)	(Ali et al., 2021), (Ram & Montibeller, 2013)
	Lack of skilled labor in the environment (C45)	(Mkwanazi, 2018)
	Technology update capability (C46)	(Gudo et al., 2020)
	Growing costs of raw material supply (C47)	(Balali et al., 2015), (Gudo et al., 2020)
	Existence of foreign investors (C48)	(Ali et al., 2021)
	Ability to compete in the market (C49)	Expert's opinion

4-1- Results of fuzzy SWARA method

As mentioned before, the final list of criteria and sub-criteria related to the evaluation of industries active in the stock market is first presented to the decision-making board (experts). This committee includes five experts in the field of capital market, who have been active in the field of university teaching for more than 10 years. In the next step, the experts determine the relative weight for the main criteria and the relevant sub-criteria. The process is such that the Board of Experts, after several rounds of discussion, formed a common consensus and arranged the main criteria from the most important criteria to the least important criteria. In the following, the relative importance of the mean value ($\tilde{S}_j$) for each of the criteria examined by experts is evaluated using a fuzzy verbal scale. Then the fuzzy coefficient $\tilde{k}_j$ for each criterion is calculated through Equation 9. As can be deduced from the results, the most important criteria belong to the resources and the ability of the organization to create a competitive advantage, followed by others.

Table 3- Local weight of main criteria

Criteria	$\tilde{S}_j$	$\tilde{k}_j$	$\tilde{q}_j$	$\tilde{w}_j$
C_1		(1, 1, 1)	(1, 1, 1)	(0.358, 0.377, 0.404)
C_3	(0.283, 0.333, 0.408)	(1.283, 1.333, 1.408)	(0.710, 0.750, 0.779)	(0.254, 0.283, 0.315)
C_2	(0.377, 0.467, 0.613)	(1.377, 1.467, 1.613)	(0.440, 0.511, 0.566)	(0.158, 0.193, 0.229)
C_4	(0.260, 0.300, 0.354)	(1.260, 1.300, 1.354)	(0.325, 0.393, 0.449)	(0.116, 0.148, 0.181)

In a similar way, the sub-criteria related to each main criterion are evaluated by the decision-making board. The local weight of each sub-criterion can be seen in the tables provided in section (d) of the appendix, respectively.

Finally, the global weights of sub-criteria are shown in Table 4. For example, the local weight of sub-criterion (C11) in its own group is equal to (0.067, 0.084, 0.101) and also the weight of criterion (C1) is equal to (0.358, 0.377, 0.404). As a result, the global weight for the C11 sub-criterion obtained by multiplying these weights is (0.024, 0.032, 0.041). In the same way, the global optimal weight for other sub-criteria is determined. As can be deduced from the results, the sub-criteria (C12) (0.135), (C13) (0.102) and (C32) (0.101) are the three main indicators for evaluating organizational strategies. In addition, (C48) is the least important of all indicators. Table 4 uses the relative weights in the fuzzy EDAS model.

Table 4- Final weight of criteria and sub-criteria

Criteria	Criteria fuzzy local weight	Sub-criteria	Sub-criteria fuzzy local weights	Global fuzzy weights	Global weights	Rank
C_1	(0.358, 0.377, 0.404)	C_{11}	(0.067, 0.084, 0.101)	(0.024, 0.032, 0.041)	0.032	13
		C_{12}	(0.342, 0.356, 0.375)	(0.122, 0.134, 0.151)	0.135	1
		C_{13}	(0.247, 0.270, 0.295)	(0.088, 0.102, 0.119)	0.102	2
		C_{14}	(0.104, 0.119, 0.135)	(0.037, 0.045, 0.055)	0.045	9
		C_{15}	(0.153, 0.171, 0.189)	(0.055, 0.064, 0.076)	0.065	5
C_2	(0.158, 0.193, 0.229)	C_{21}	(0.314, 0.334, 0.363)	(0.049, 0.064, 0.083)	0.065	6
		C_{22}	(0.153, 0.183, 0.216)	(0.024, 0.035, 0.049)	0.036	11
		C_{23}	(0.103, 0.133, 0.164)	(0.016, 0.026, 0.037)	0.026	16

		C_{24}	(0.073, 0.099, 0.128)	(0.012, 0.019, 0.029)	0.020	18
		C_{25}	(0.223, 0.251, 0.283)	(0.035, 0.048, 0.065)	0.049	8
C_3	(0.254, 0.283, 0.315)	C_{31}	(0.224, 0.255, 0.290)	(0.057, 0.072, 0.091)	0.073	4
		C_{32}	(0.333, 0.353, 0.383)	(0.085, 0.100, 0.121)	0.101	3
		C_{33}	(0.097, 0.124, 0.153)	(0.025, 0.035, 0.048)	0.035	12
		C_{34}	(0.147, 0.174, 0.203)	(0.037, 0.049, 0.064)	0.050	7
		C_{35}	(0.069, 0.093, 0.119)	(0.018, 0.026, 0.037)	0.027	15
C_4	(0.116, 0.148, 0.181)	C_{41}	(0.261, 0.290, 0.332)	(0.030, 0.043, 0.060)	0.044	10
		C_{42}	(0.170, 0.204, 0.247)	(0.020, 0.030, 0.045)	0.031	14
		C_{43}	(0.082, 0.111, 0.146)	(0.010, 0.016, 0.026)	0.017	19
		C_{44}	(0.119, 0.151, 0.191)	(0.014, 0.022, 0.035)	0.023	17
		C_{45}	(0.055, 0.080, 0.111)	(0.006, 0.012, 0.020)	0.012	20
		C_{46}	(0.039, 0.060, 0.086)	(0.005, 0.009, 0.016)	0.009	21
		C_{47}	(0.018, 0.032, 0.050)	(0.002, 0.005, 0.009)	0.005	23
		C_{48}	(0.014, 0.025, 0.041)	(0.002, 0.004, 0.007)	0.004	24
		C_{49}	(0.029, 0.046, 0.068)	(0.003, 0.007, 0.012)	0.007	22

4-2- Results of fuzzy EDAS method

In this section, the results of the implementation of the fuzzy EDAS method are expressed. Initially, each decision-maker presents his or her mental preferences for evaluating each alternative over each criterion using defined verbal expressions. As mentioned before, the alternatives of this research include 6 different industry categories as follows.

Alternative 1) Basic metals

Alternative 2) Chemical products

Alternative 3) Investment

Alternative 4) Extraction of metal ores

Alternative 5) Financing papers

Alternative 6) Insurance and pension fund including social security

Using the results of the previous steps as well as applying the equations related to the prioritization method, the matrices of positive and negative distances are averaged based on the following tables. For these calculations, a set of utility and non-utility criteria must first be determined. For this purpose, the weighted sum of positive and negative distances of each alternative ($\widetilde{sp}_i$ ، $\widetilde{sn}_i$) is obtained. Then their normalized values ($\widetilde{nsp}_i$ ، $\widetilde{nsn}_i$) as well as the fuzzy evaluation score ($\widetilde{as}_i$) of all alternatives are calculated. It is worth noting that the best non-fuzzy $\widetilde{as}_i$ performance is also obtained by applying the graded averaging method to the integrated display. Based on the results obtained, alternative A_2 has the highest evaluation score and is ranked first. In general, the final priority of the options is $A_2 \succ A_5 \succ A_6 \succ A_1 \succ A_3 \succ A_4$. Details of numerical calculations of the fuzzy EDAS method are available in section € of the appendix.

Table 5 – Total weight of distance and final weight

	$\widetilde{sp}_i$	$\widetilde{sn}_i$	$\widetilde{nsp}_i$	$\widetilde{nsn}_i$	$\widetilde{as}_i$	$k(\widetilde{as}_i)$	Rank
A_1	(-0.103,0.043,0.19)	(-0.093,0.063,0.223)	(-0.541,0.225,1.003)	(-0.797,0.491,1.748)	(-0.669,0.358,1.375)	0.357	4
A_2	(-0.094,0.192,0.463)	(0.007,0.015,0.022)	(-0.493,1.013,2.439)	(0.821,0.882,0.943)	(0.164,0.948,1.691)	0.941	1
A_3	(-0.071,0.057,0.186)	(-0.064,0.11,0.28)	(-0.374,0.299,0.98)	(-1.26,0.114,1.516)	(-0.817,0.207,1.248)	0.210	5
A_4	(-0.067,0.047,0.161)	(-0.03,0.125,0.273)	(-0.353,0.246,0.846)	(-1.206,-0.008,1.239)	(-0.779,0.119,1.043)	0.123	6
A_5	(-0.07,0.051,0.172)	(-0.118,0.065,0.245)	(-0.369,0.269,0.905)	(-0.975,0.478,1.952)	(-0.672,0.373,1.429)	0.375	2
A_6	(-0.066,0.061,0.187)	(-0.114,0.073,0.256)	(-0.348,0.321,0.988)	(-1.07,0.41,1.918)	(-0.709,0.366,1.453)	0.368	3

4-3- Sensitivity analysis

In this section, sensitivity analysis is performed in order to monitor the stability of the results in accordance with the instructions presented in the article)Kahraman, 2002(. The purpose of analyzing the proposed fuzzy SWARA-fuzzy EDAS decision model is to generate new weight vectors and investigate their effect on changes in the ranking of alternatives. New weight coefficients are calculated based on changes in the most effective criterion (sensitive criterion). In the following, the weight ratios of other criteria are concluded according to the proportions of the weights in the sensitivity analysis process. New sets of weight vectors in the scenarios are also created with respect to the elastic weight coefficient, so that the relative compensation of other values of the weight coefficients in relation to the given changes in weight explains the most important criterion)Behzad, Zolfani, Pamucar, & Behzad, 2020(. Based on what was described above, the elastic weight coefficient for criterion C12 has been estimated and the range of changes in criterion C12 weight coefficient has also been obtained. Threshold values for C12 criterion are calculated as intervals [-0.135, 0.878]. After defining the limit values of C12 criterion, the new weight coefficient vectors for 15 scenarios are obtained according to the table below.

Table 6- Weights of criteria based on each scenario

	w_{S_1}	w_{S_2}	w_{S_3}	w_{S_4}	w_{S_5}	w_{S_6}	w_{S_7}	w_{S_8}	w_{S_9}	$w_{S_{10}}$	$w_{S_{11}}$	$w_{S_{12}}$	$w_{S_{13}}$	$w_{S_{14}}$	$w_{S_{15}}$
C_{11}	0.037	0.035	0.033	0.032	0.030	0.028	0.026	0.024	0.022	0.021	0.019	0.017	0.015	0.013	0.012
C_{12}	0.000	0.050	0.099	0.149	0.198	0.248	0.297	0.347	0.396	0.446	0.495	0.545	0.594	0.644	0.693
C_{13}	0.118	0.112	0.106	0.100	0.095	0.089	0.083	0.077	0.072	0.066	0.060	0.054	0.049	0.043	0.037
C_{14}	0.052	0.049	0.047	0.044	0.042	0.039	0.037	0.034	0.032	0.029	0.027	0.024	0.021	0.019	0.016
C_{15}	0.075	0.071	0.068	0.064	0.060	0.057	0.053	0.049	0.046	0.042	0.038	0.035	0.031	0.027	0.024
C_{21}	0.075	0.071	0.068	0.064	0.060	0.057	0.053	0.049	0.046	0.042	0.038	0.035	0.031	0.027	0.024
C_{22}	0.042	0.040	0.037	0.035	0.033	0.031	0.029	0.027	0.025	0.023	0.021	0.019	0.017	0.015	0.013
C_{23}	0.030	0.029	0.027	0.026	0.024	0.023	0.021	0.020	0.018	0.017	0.015	0.014	0.012	0.011	0.009
C_{24}	0.023	0.022	0.021	0.020	0.019	0.017	0.016	0.015	0.014	0.013	0.012	0.011	0.010	0.008	0.007
C_{25}	0.057	0.054	0.051	0.048	0.045	0.043	0.040	0.037	0.034	0.032	0.029	0.026	0.023	0.021	0.018
C_{31}	0.084	0.080	0.076	0.072	0.068	0.064	0.060	0.055	0.051	0.047	0.043	0.039	0.035	0.031	0.027
C_{32}	0.117	0.111	0.105	0.099	0.094	0.088	0.082	0.077	0.071	0.065	0.060	0.054	0.048	0.042	0.037
C_{33}	0.040	0.038	0.036	0.034	0.032	0.031	0.029	0.027	0.025	0.023	0.021	0.019	0.017	0.015	0.013
C_{34}	0.058	0.055	0.052	0.049	0.046	0.044	0.041	0.038	0.035	0.032	0.029	0.027	0.024	0.021	0.018
C_{35}	0.031	0.030	0.028	0.027	0.025	0.024	0.022	0.020	0.019	0.017	0.016	0.014	0.013	0.011	0.010
C_{41}	0.051	0.048	0.046	0.043	0.041	0.038	0.036	0.033	0.031	0.028	0.026	0.023	0.021	0.018	0.016
C_{42}	0.036	0.034	0.032	0.031	0.029	0.027	0.025	0.024	0.022	0.020	0.018	0.017	0.015	0.013	0.011

C_{43}	0.020	0.019	0.018	0.017	0.016	0.015	0.014	0.013	0.012	0.011	0.010	0.009	0.008	0.007	0.006
C_{44}	0.027	0.025	0.024	0.023	0.021	0.020	0.019	0.017	0.016	0.015	0.014	0.012	0.011	0.010	0.008
C_{45}	0.014	0.013	0.012	0.012	0.011	0.010	0.010	0.009	0.008	0.008	0.007	0.006	0.006	0.005	0.004
C_{46}	0.010	0.010	0.009	0.009	0.008	0.008	0.007	0.007	0.006	0.006	0.005	0.005	0.004	0.004	0.003
C_{47}	0.006	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.002	0.002	0.002
C_{48}	0.005	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.001
C_{49}	0.008	0.008	0.007	0.007	0.006	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.003	0.003	0.003

According to the obtained results presented in Table 6, when the weight of criterion C12 changes, no significant change occurred in the final rank of option A2, and in all scenarios A2 remains the dominant alternative. Therefore, it can be concluded that the final result for choosing the best industry among the six available alternatives is so robust against changing the most important criterion's weight. However, the final rank of other alternatives is so sensitive to changing the most important criterion's weight. Therefore, gaining the weight of each criterion logically and scientifically plays an important role in choosing the optimal industry.

5- Numerical results of the optimization phase

After prioritizing the industries active in the capital market by a multi-criteria decision model, in this section, the amount of investment in each alternative is solved optimally.

5-1- Determination of the input parameters

The objective functions of the mathematical model include maximizing the priority of each company based on market value, maximizing the revenue-to-profit ratio, and ultimately minimizing risk. Therefore, determining the parameters related to each of the objective functions is controversial. In this study, the priority of each potential company for investment based on relation (32) is obtained.

$$P_i = \frac{Value_i}{\sum_{i=1} Value_i} \qquad \forall i \in potential\ firms \qquad (32)$$

Another important challenge is to determine the revenue and cost parameters in order to calculate the value of the second objective function. This information is available separately on the *Codal website* and can be extracted directly for each company. Finally, in order to determine the amount of investment risk in each company, the data available on the *Codal website* are used in relation to the adjusted price with the increase of capital and also the adjusted price with the increase of capital and cash profit. However, this data is limited to the price adjusted by increasing capital and cash dividends and does not yield returns. Therefore, it is necessary to calculate the return of each company in each time period through the following equation.

$$Return_t = \frac{A_t}{A_{t-1}} \times 100$$

Where A_t represents the adjusted price with an increase in capital and cash dividend in year *t*. To calculate the risk, it is sufficient to calculate the standard variance of returns for each company. After performing the necessary calculations in the Excel software environment, the final data related to each company is available in the following table. After solving the mathematical model, the input parameters of the Pareto front are presented as figure 2.

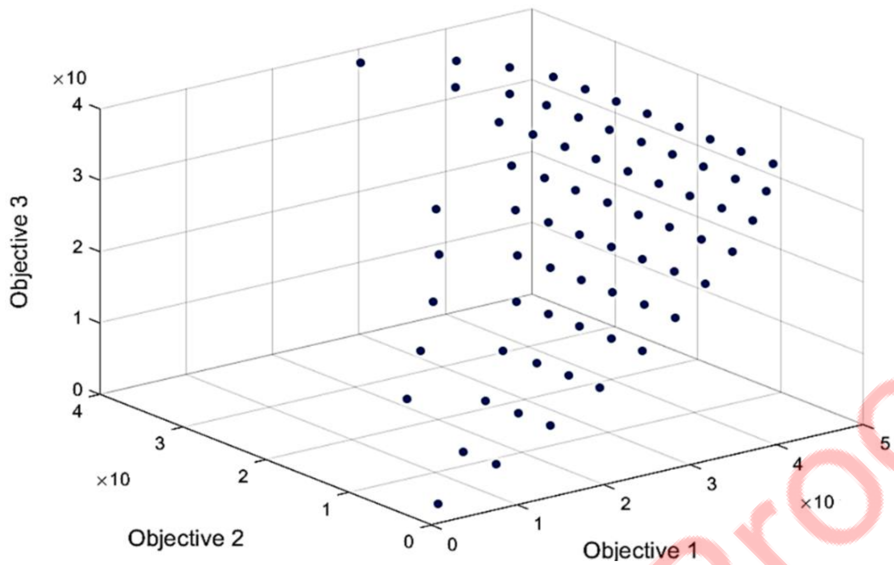

Figure 2- Pareto front resulting from solving the mathematical model

According to Figure 2, it can be seen that the produced Pareto front has 71 members, which changes from 22.284 to 402.982 for the first objective function, 15.475 to 391.33 for the second objective function, and 15.579 to 412. 38 for the third objective function. Also, the corner points of the Pareto front include (40.98,15.47,38.41), (29.56,39.33,38.41) and (22.28,15.47,16.57), in each of which one of the objective functions is at its best. One of the most important problems in solving multi-objective problems is choosing a Pareto member as the final answer to implement in real world conditions; since the members of the produced front are non-dominated and have no superiority over each other. In this study, in order to solve this problem, a method for calculating the level of efficiency of each Pareto member based on proximity to the ideal solution (the solution in which all objective functions have the best value) is presented.

5-2- Selection of the best performing Pareto member

In this method, first the mathematical model is solved for each of the objective functions and the optimal value of the objective functions is calculated separately. Then the Euclidean distance of each Pareto member to the ideal point is calculated and the Pareto member with the shortest distance to the ideal point is selected as the final answer. The steps of this method include as follows.

Step 1: Solve the mathematical model for each of the objective functions separately and store the optimal values in Z_1^* ، Z_2^* and Z_3^*

Step 2: Solve the mathematical model using the Epsilon constraint method and store the solutions in the optimal set PS^*

Step 3: Calculate the Euclidean distance of the members of the set PS^* with (Z_1^*, Z_2^*, Z_3^*) based on Equation (80) and produce the MID set

Step 4: Select the Pareto member with the lowest MID value as the final solution

The following equation for calculating MID is presented as follows.

$$MID_i = \sqrt{\sum_{j=1}^{n_{obj}} \left(Z_j^* - Z_{ij}\right)^2} \qquad i \in PS^* \qquad 33$$

Where n_{obj} equals the number of objective functions and Z_{ij} equals the value of the j function for the i Pareto member. Based on this relationship, a Pareto member with the highest efficiency can be selected. After performing numerical calculations to calculate MID_i, the best Pareto member, with the first objective function value of Z_1^{MID} = 32.99, the second objective function value of Z_2^{MID} =22.41 and the third objective function value of Z_3^{MID} = 23.71, has a value of MID = 279.02. In this solution, the optimal amount of investment in each company is as follows.

Table 7- Optimal amount of investment (percentage) in each company

Code	Investment percentage	Code	Investment percentage
Shrak1	0.015	Petrol1	0.020
Parsan1	0.063	Jem Pilen1	0.012
Shefen1	0.022	Khorasan1	0.010
Kermasha1	0.010	Noori1	0.042
Shekhark1	0.020	Pars1	0.069
Shapdis1	0.045	Pakshoo1	0.023
Shiraz1	0.021	Jam1	0.049
Shiran1	0.020	Fars1	0.220
Buali1	0.011	Tapko1	0.076
Gharn1	0.005	Shghadir1	0.007
Shegooya1	0.017	Aria1	0.059
Shekabir1	0.024	Maroon1	0.093
Shelord1	0.005	Zagros1	0.039
Shejem1	0.005		

According to the information in Table 7, 27 companies have been selected for investment, which according to the constraints of the mathematical model, is less than 30 and is completely justified. Figure 3 graphically shows the optimal amount of investment in each company.

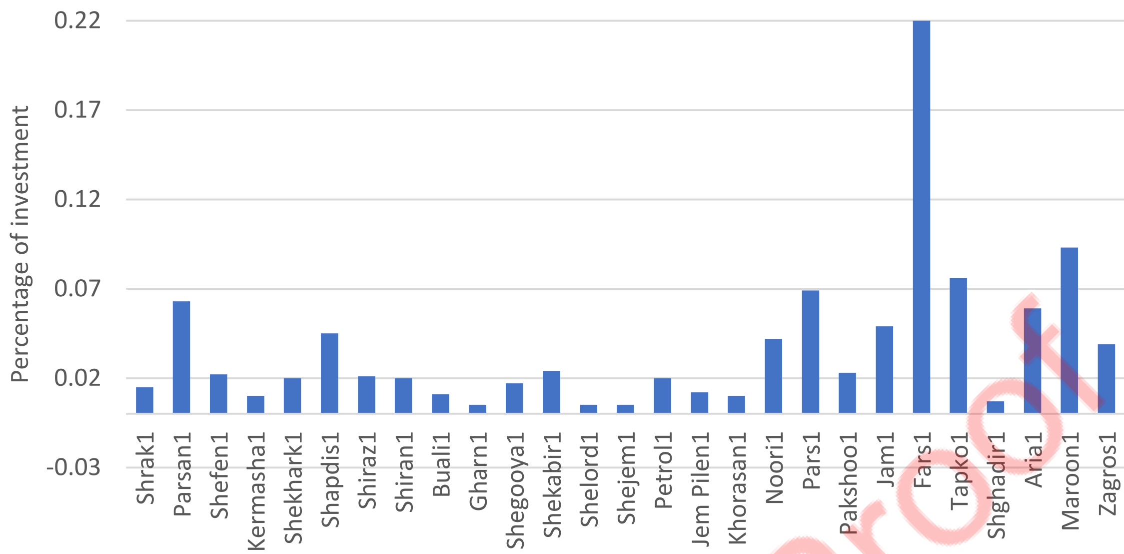

Figure 3- Percentage of investment in each company

As can be seen, the highest share of investment is related to Fars 1 with a value of 22.2% and the lowest amount is related to the Gharn1 with a value of 0.05%.

5-3- Numerical analysis in uncertainty conditions

In this section, in order to investigate the sensitivity level of the proposed model to the uncertainty of input parameters, different combinations of robustness parameters are considered and the Pareto member is determined with the best MID value for each combination.

Table 8- the sensitivity of the mathematical model to changes of the robustness parameters

Instance	Γ_1	Γ_2	Γ_3	$\min_{i\in PS^*}(MID_i)$	Z_1^{MID}	Z_2^{MID}	Z_3^{MID}
1	5	20	34	245.659	40.2242	1.54728	38.41338
2	8	18	32	278.5501	34.19463	5.32591	38.41338
3	10	16	30	293.6505	29.72421	9.10453	38.41338
4	12	14	28	233.4255	25.91092	12.88316	38.41338
5	14	12	26	244.9353	20.97057	16.66178	38.41338
6	16	10	24	280.9324	15.22036	20.44041	38.41338
7	18	8	22	279.6252	11.76803	24.21903	34.73775
8	20	5	20	286.1519	9.59114	27.99765	31.06212
9	22	34	18	282.2125	9.02072	31.77628	27.3865
10	24	32	16	264.2044	8.81535	18.47125	20.03525
11	26	30	14	257.1233	8.81535	20.34714	16.35962
12	28	28	12	249.8394	8.81535	23.65418	12.684
13	30	26	10	245.6434	8.81535	24.98786	9.00837
14	32	24	8	235.7195	8.81535	26.84133	5.33274
15	34	22	5	256.1065	8.81535	27.4411	1.65712

According to Table 8, it can be seen that the value of $\min_{i\in PS^*}(MID_i)$ varied in the range of 233.42 to 293.65, which indicates the dispersion level of 19.73% of Pareto members in the optimal space based on different robustness parameters values. In fact, changes in levels of uncertainty in the model cause the Pareto set to change by about 20%, which is a high amount for strategic level decisions and requires managers to pay attention to increasing the accuracy in determining the exact amount of input parameters. In addition, it can be seen that by changing Γ_1 from small to large values, it causes the first objective function in the most efficient Pareto member to be in descending order. However, in $\Gamma_1 > 10$ the changes are eliminated and the value of the first objective function is fixed. The third objective function also has an ascending trend with the descending changes of parameter Γ_3. This indicates that the higher the level of uncertainty, the lower the quality of the solutions generated, and managers must develop tools to predict the input data. The following figure shows the sensitivity of different objective functions to changing robustness parameters.

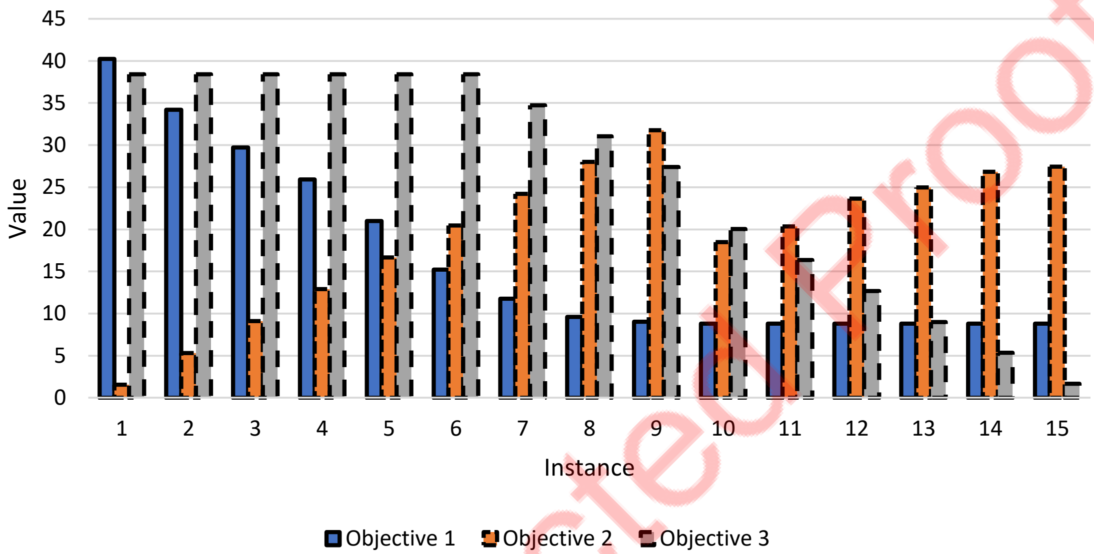

Figure 4. Sensitivity of the triple objective functions to the robustness parameters

According to Figure 4, the first objective function has a downward trend by changing the robustness parameter in the range $5 < \Gamma_1 < 20$, which indicates the negative effect of increasing the level of uncertainty in obtaining the final solutions. But for $\Gamma_1 > 22$ the value of the objective function has not changed, which indicates the creation of bad conditions in the model for the first objective function. In fact, the sensitivity threshold of the first objective function is equal to $\Gamma_1 = 22$. Changes in the robustness parameter for the third objective function in the range of $24 < \Gamma_3 < 34$ did not cause any changes in its value, which indicates the sensitivity threshold Γ_3 = 24 for this objective function. But at values $5 < \Gamma_3 < 22$ by decreasing the value of this parameter, the value of the third objective function decreases because the model is able to produce higher quality solution for this objective function. Regarding the sensitivity of the second objective function, it can be said that it behaves similarly to the first function. In fact, by increasing the value of the robustness parameter, the model produces lower quality solutions and the value of this objective function decreases. But if the level of uncertainty decreases, the value of this objective function also increases ascending and higher quality solutions are obtained. As can be observed, the combination $\Gamma_1 = 12$, $\Gamma_2 = 14$ and $\Gamma_3 = 28$ provides the best value of MID for the Pareto members produced in different cases where $Z_1^{MID} = 25.910$, $Z_2^{MID} = 12.883$ and $Z_3^{MID} = 38.413$.

According to the obtained results, some details and explanations of the implications related to paper are as follow. First, the proposed hybrid approach of fuzzy-MCDM and optimization models under interval uncertainty can be applied by investment managers in the Iranian capital market to optimize their investment decisions. By considering multiple criteria and alternative evaluations, this approach can help them make more informed decisions that better reflect their investment goals. Moreover, fuzzy-SWARA and fuzzy-EDAS methods have demonstrated their usefulness in the context of investment management. However, their application can be extended to other areas beyond investment management, such as project management or risk assessment, where decision-making is complex and uncertain. In addition, multi-objective optimization models that incorporate interval uncertainty are relevant in many contexts. The proposed model in the paper can be adapted and applied in other fields, such as supply chain management, environmental management, or public policy, where trade-offs between multiple objectives and interval uncertainty are relevant. the managerial analysis and decision-making policies resulting from the proposed approach can be useful for policymakers and investors. The study's findings can help guide the development of investment strategies that balance risk and return, which can be used to inform investment policies and attract foreign investors. Finally, the study's focus on the Iranian capital market highlights the importance of considering regional or country-specific factors when designing investment management approaches. As such, this approach can be valuable for other researchers or practitioners working in other emerging markets or developing economies, as it emphasizes the need to consider context-specific factors when designing investment management approaches.

6- Conclusion

In this research, a hybrid approach based on multi-criteria decision making and mathematical optimization is proposed to investigate investment management problem in stock market in Iran. for this purpose, some active industrial companies are evaluated using a set of criteria and sub-criteria extracted from the literature. Using historical financial data, a mathematical model is designed to optimize the amount of investment in each of these companies. Finally, due to the fact that it is difficult to determine the exact value of some input parameters, a robust programming method to face interval uncertainty have been developed. Based on the obtained results, the sub-criteria (C12) with global weight equal to (0.135), (C13) with global weight equal to (0.102) and (C32) with global weight equal to (0.101) are selected as the highest score criteria to evaluate the alternatives. The prioritization of industries also shows that the chemical industry has the highest priority for investment. After solving the multi-objective optimization model in deterministic condition, it is observed that the generated Pareto front has 71 members with a boundary in the range of (22.284-402.982) for the first objective function, (15.475-39.331) for the second objective function and (15.579-38.412) for the third objective function. Also, the corner points of the Pareto front include (40.98,15.47,38.41), (29.56,39.33,38.41) and (22.28,15.47,16.57), in each of which one of the objective functions is at its best.

It should be noted that, one of the most important problems in solving multi-objective problems is choosing one of the Pareto members as the final solution to implement in real world conditions. In this study, in order to solve this problem, a heuristic method is developed for calculating the efficiency of each Pareto member based the ideal solution. After performing numerical calculations to calculate MID_i, the best Pareto member, with the first objective function value of $Z_1^{MID} = 32.99$, the second objective function value of Z_2^{MID} =22.41 and the third objective function value of Z_3^{MID} = 23.71, has MID = 279.02. In the selected optimal solution, 27 companies were selected for investment, which according to the constraints of the mathematical model, is less than 30 and is completely justified. The highest share of investment is related to Fars 1 with a value of 22.2% and the lowest amount is related to the Gharn1 with a value of 0.05%. In solving the model under conditions of uncertainty, it is observed that the value of $\min_{i \in PS^*}(MID_i)$ varies in the range of 233.42 to 293.65, which indicates the level of dispersion of 19.73% of Pareto members based on different values of robustness parameters. In fact, changes in uncertainty levels in the model cause the Pareto set to change by about 20%, which is a high amount for strategic level decisions and requires managers to pay more attention to determine the exact amount of input parameters. In addition, it can be seen that by changing Γ_1 from small to large values, it causes the first objective function in the most efficient Pareto member to be in descending order. However, in $\Gamma_1 > 10$ the changes are eliminated and the value of the first objective function is fixed. The third objective function also has an ascending trend with the descending changes of parameter Γ_3. This indicates that the higher the level of uncertainty, the lower the quality of the solutions generated, and managers must develop tools to predict the input data.

7- References

Ali, E. B., Agyekum, E. B., & Adadi, P. (2021). Agriculture for Sustainable Development: A SWOT-AHP Assessment of Ghana's Planting for Food and Jobs Initiative. *Sustainability, 13*(2), 628.

Allcott, H., & Keniston, D. (2018). Dutch disease or agglomeration? The local economic effects of natural resource booms in modern America. *The Review of Economic Studies, 85*(2), 695-731.

Ansari, Z. N., Kant, R., & Shankar, R. (2020). Evaluation and ranking of solutions to mitigate sustainable remanufacturing supply chain risks: a hybrid fuzzy SWARA-fuzzy COPRAS framework approach. *International Journal of Sustainable Engineering, 13*(6), 473-494.

Arif, A., Sadiq, M., Shabbir, M. S., Yahya, G., Zamir, A., & Bares Lopez, L. (2020). The role of globalization in financial development, trade openness and sustainable environmental-economic growth: evidence from selected South Asian economies. *Journal of Sustainable Finance & Investment*, 1-18.

Balali, H., Saadi, H., & Ghazvineh, S. (2015). Investment prioritizing of food and agricultural processing industries (case study: Hamedan County). *Journal of Research and Rural Planning, 4*(1), 149-159.

Banaeian, N., Mobli, H., Fahimnia, B., Nielsen, I. E., & Omid, M. (2018). Green supplier selection using fuzzy group decision making methods: A case study from the agri-food industry. *Computers & Operations Research, 89*, 337-347.

Behzad, M., Zolfani, S. H., Pamucar, D., & Behzad, M. (2020). A comparative assessment of solid waste management performance in the Nordic countries based on BWM-EDAS. *Journal of Cleaner Production, 266*, 122008.

Bertsimas, D., & Sim, M. (2004). The price of robustness. *Operations research, 52*(1), 35-53.

Caplan, L., Griswold, J. S., & Jarvis, W. F. (2013). From SRI to ESG: The Changing World of Responsible Investing. *Commonfund Institute*.

Darmian, S. M., & Farughi, H. (2022). An optimization-based approach for managing downstream logistic operations: A real application in pandemic outbreak conditions. *Computers & Industrial Engineering, 171*, 108495.

Darmian, S. M., Fattahi, M., & Keyvanshokooh, E. (2021). An optimization-based approach for the healthcare districting under uncertainty. *Computers & Operations Research, 135*, 105425.

Doaei, M., & saberfard, M. (2021). A chance constrained recourse approach for the portfolio selection problem in Iran capital market. Financial Engineering and Portfolio Management, 12(46), 667-690.

Doaei, M., Mirzaei, S. A., & Rafigh, M. (2021). Hybrid Multilayer Perceptron Neural Network with Grey Wolf Optimization for Predicting Stock Market Index. Advances in Mathematical Finance and Applications, 6(4), 883-894. doi: 10.22034/amfa.2021.1903474.1452

Farughi, H., Dolatabadiaa, M., Moradi, V., Karbasi, V., & Mostafayi, S. (2017). Minimizing the number of tool switches in flexible manufacturing cells subject to tools reliability using genetic algorithm. *Journal of Industrial and Systems Engineering, 10*(special issue on Quality Control and Reliability), 17-33.

Farughi, H., & Mostafayi, S. (2016). Robust Optimization Approach for Design for a Dynamic Cell Formation Considering Labor Utilization: Bi-objective Mathematical Model. *International Journal of Supply and Operations Management, 3*(1), 1143-1167.

Farughi, H., & Mostafayi, S. (2017). A hybrid approach based on ANP, ELECTRE and SIMANP metaheuristic method for outsourcing manufacturing procedures according to supply chain risks-Case study: A medical equipment manufacturer company in Iran. *Decision Science Letters, 6*(1), 77-94.

Ferneini, E. M. (2020). The financial impact of COVID-19 on our practice. *Journal of Oral and Maxillofacial Surgery, 78*(7), 1047-1048.

Ghorabaee, M. K., Amiri, M., Zavadskas, E. K., & Antucheviciene, J. (2018). A new hybrid fuzzy MCDM approach for evaluation of construction equipment with sustainability considerations. *Archives of Civil and Mechanical Engineering, 18*, 32-49.

Ghorabaee, M. K., Zavadskas, E. K., Amiri, M., & Turskis, Z. (2016). Extended EDAS method for fuzzy multi-criteria decision-making: an application to supplier selection. *International journal of computers communications & control, 11*(3), 358-371.

Gudo, A. J. A., Deng, J., Belete, M., & Abubakar, G. A. (2020). Estimation of Small Onshore Wind Power Development for Poverty Reduction in Jubek State, South Sudan, Africa. *Sustainability, 12*(4), 1483.

Ho, W.-R. J., Tsai, C.-L., Tzeng, G.-H., & Fang, S.-K. (2011). Combined DEMATEL technique with a novel MCDM model for exploring portfolio selection based on CAPM. *Expert systems with applications, 38*(1), 16-25.

Kahraman, Y. R. (2002). *Robust sensitivity analysis for multi-attribute deterministic hierarchical value models*. Retrieved from

Keršuliene, V., Zavadskas, E. K., & Turskis, Z. (2010). Selection of rational dispute resolution method by applying new step-wise weight assessment ratio analysis (SWARA). *Journal of business economics and management, 11*(2), 243-258.

Keshavarz Ghorabaee, M., Zavadskas, E. K., Olfat, L., & Turskis, Z. (2015). Multi-criteria inventory classification using a new method of evaluation based on distance from average solution (EDAS). *Informatica, 26*(3), 435-451.

Khodaverdizadeh, M., & Mohammadi, S. (2016). Determination of Market Structure and Ranking Target Markets of Iran's Export of Medicinal Plants. *Journal of Applied Economics Studies in Iran, 5*(20), 201-220.

Kisman, Z., & Krisandi, D. (2019). How to predict financial distress in the wholesale sector: Lesson from Indonesian Stock Exchange. *Journal of Economics and Business, 2*(3).

Kou, G., Ergu, D., Lin, C., & Chen, Y. (2016). Pairwise comparison matrix in multiple criteria decision making. *Technological and economic development of economy, 22*(5), 738-765.

Kou, G., Peng, Y., & Wang, G. (2014). Evaluation of clustering algorithms for financial risk analysis using MCDM methods. *Information Sciences, 275*, 1-12.

Li, N., & Zhao, H. (2016). Performance evaluation of eco-industrial thermal power plants by using fuzzy GRA-VIKOR and combination weighting techniques. *Journal of Cleaner Production, 135*, 169-183.

Liza, M., & Morales Anaya, J. A. (2018). *Export boom and economic performance: Bolivia 2004-2015*. Retrieved from

Mansley, M. (2000). *Socially responsible investment: A guide for pension funds and institutional investors*: Monitor press.

Mardani, A., Nilashi, M., Zakuan, N., Loganathan, N., Soheilirad, S., Saman, M. Z. M., & Ibrahim, O. (2017). A systematic review and meta-Analysis of SWARA and WASPAS methods: Theory and applications with recent fuzzy developments. *Applied Soft Computing, 57*, 265-292.

Mkwanazi, S. (2018). Evaluating factors of government investment in new industrial parks. *Journal of Emerging Trends in Economics and Management Sciences*, *9*(6), 350-355.

Mostafayi Darmian, S., Doaei, M. (2022). Optimization of Stock Portfolio Selection in Iran Capital Market Using Meta-heuristic Algorithms. Quarterly Journal of Applied Theories of Economics, 8(4), 253-284. doi: 10.22034/ecoj.2022.47049.2913

Polat, G., & Bayhan, H. G. (2020). Selection of HVAC-AHU system supplier with environmental considerations using Fuzzy EDAS method. *International Journal of Construction Management*, 1-9.

Popkova, E. G., Bogoviz, A. V., Ragulina, Y. V., & Alekseev, A. N. (2018). Perspective model of activation of economic growth in modern Russia *Management of Changes in Socio-Economic Systems* (pp. 171-177): Springer.

Rais, M., Acharya, S., & Sharma, N. (2013). Food processing industry in India: S&T capability, skills and employment opportunities. *J Food Process Technol, 4*(9), 1-13.

Ram, C., & Montibeller, G. (2013). Exploring the impact of evaluating strategic options in a scenario-based multi-criteria framework. *Technological Forecasting and Social Change, 80*(4), 657-672.

Rezaei, J. (2015). Best-worst multi-criteria decision-making method. *Omega, 53*, 49-57.

Rezaei, J. (2016). Best-worst multi-criteria decision-making method: Some properties and a linear model. *Omega, 64*, 126-130.

Saffarian, M., Mostafayi, S., Kazemi, S. M., & Niksirat, M. (2021). A two-level pricing-inventory-routing problem in green Closed-loop supply chain: Bi-level programming and heuristic method. *Journal of Industrial and Systems Engineering*, *13*(4), 62-80.

Shabbir, M. S., & Wisdom, O. (2020). The relationship between corporate social responsibility, environmental investments and financial performance: evidence from manufacturing companies. *Environmental Science and Pollution Research*, 1-12.

Soyster, A. L. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations Research, 21*(5), 1154-1157.

Stević, Ž., Vasiljević, M., Zavadskas, E. K., Sremac, S., & Turskis, Z. (2018). Selection of carpenter manufacturer using fuzzy EDAS method. *Engineering Economics, 29*(3), 281-290.

Talan, G., & Sharma, G. D. (2019). Doing well by doing good: A systematic review and research agenda for sustainable investment. *Sustainability, 11*(2), 353.

Thoumi, F. E. (2009). Necessary, sufficient and contributory factors generating illegal economic activity, and specifically drug-related activity, in Colombia. *Iberoamericana (2001-), 9*(35), 105-126.