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Wavelet Analysis of Stock Returns and Total Index with Moving Average of Stock Returns and Total Index

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Abstract

This research aims to investigate and analyze the behavior patterns of stock market fluctuations so that appropriate strategies with varying time horizons can be determined based on the characteristics extracted from different time layers and the level of economic activity Measured by the investors. This research investigates and analyses stock market fluctuations in different periods by applying discrete wavelet transformation with maximum overlap in MATLAB software. For this purpose, the variances of effective indicators are compared and analyzed during 2011-2023. This research shows that the wavelet variance of the stock return is more than the moving average of the stock return. According to the movement scales of each stock return and the moving average of the stock returns during the long-term scales, the wavelet variance is less and the comovement is less. Still, during the short-term time scales, the comovement is greater, and the variance of the wavelet return is greater among them. The variance of the moving average of the total index is greater than that of the total index. According to the movement scales, each of the total index and the moving average of the total index has less wavelet variance and less comovement during the long-term scales. Still, the commitment has increased during the short-term time scales, and the wavelet return variance is higher.

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1 Introduction

Studying the behavior of returns and volatility in the return of securities and analyzing them necessitates the discovery of a behavioral pattern of stock returns. If this pattern is discovered, shareholders can select the best stocks by evaluating their and other stocks in the market. As a result, they can hold, sell, or replace stocks.

Consequently, the temporal stability and linearity of the relationships between variables can be supposed when over changes within the company such as change of management, change of production lines, change in the composition of a company's inputs (manpower, capital, and the like), changes in economic conditions, changes in tastes, changes in government policies, increasing competition in the relevant industry, etc. should not occur in the business unit. While such changes occur continuously throughout the life of a company, it is usually to be expected. Determining the trend of time changes in the rate of return and variance of returns can be helpful in accurately estimating them and thus promoting more accurate decisions for investors. These issues lead us to use more relevant and adaptable methods to accurately calculate financial variables. For this reason, one of the most accurate ways to predict variables is to calculate them in the context of different time scales and compare the results of each scale with another scale. Consequently, in this study, the objective is to calculate the variables in the framework of different time scales by using a new and innovative tool called wavelet analysis and more clearly describe the relationships between variables, determine the optimal period of holding stocks in the Tehran Stock Exchange and also changes in variables to use them in economic decisions by investors in the stock market. In various fields of financial analysis and investment management, the calculation of the rate of return and variance of return is based on a time scale. In contrast, this calculation method leads to errors in professional and scientific decisions and analysis. In previous studies, due to the limited time scale in recognizing the real and dynamic relationship between returns and stock market volatility in different industries, the main issue in this study is to determine the real and dynamic relationship between returns and market volatility of different industries in different time horizons so that the key results can be used to measure the predictive power of returns and stock market volatility in determining the level of economic activity of investors. Accurate measurement of these relationships helps investors predict stock market movements in the future.

2 Literature review

The main objective of creating capital markets is to collect micro and macro capital and invest them in various industries to increase production and economic growth. On the other hand, the level of economic activity of investors (with the purpose of business or investment) is particularly important in the country's economic growth. Investors and capital market participants need to analyze reliable and relevant information to predict other investors' performance and the market's future trends.

Based on the discounted cash flow valuation model, companies' stock prices reflect investors' expectations of companies' future profitability. So, in general, if investor expectations are moderate, stock prices should provide information about future economic conditions as the company's interests are fully related to the level of economic activity.

Several applications of wavelet analysis in economics and finance have been proposed by Ramsey (2002), Kim and In (2003), etc.. Still, no research has been done on applying this tool to wavelet analysis of the relationship between stock market returns and the level of economic activity.

Developing countries, including Iran, have high volatility in stock prices, creating an uncertain environment for investors. Over the past decade, the stock market has played a greater role in the

Iranian economy and has experienced much volatility, as well as explaining the comovement of macroeconomic variables such as production, consumption, and investment in Iran with volatility in stock prices due to the predictive nature of prices, determine the stock market as one of the predictors of trading cycles. In addition, if the information reflected in stock prices is of high quality, these prices can also provide an accurate forecast (Shahrabadi and Bashiri, 2021).

Economics and financial phenomena may exhibit different characteristics at different time scales, so wavelet analysis tools can examine the multiple characteristics of these phenomena. Wavelet analysis is suitable for identifying periodic and seasonal patterns, structural failures, and trends, as well as multivariate analysis. This tool is used to study the behavior of unstable financial time series in the framework of different time horizons, simultaneously analyze the time and scale of financial data, and make it possible to calculate pairwise correlations in different time horizons. Wavelet analysis is a powerful tool for quantifying time series data at different time horizons quantitatively, and without losing any decision-making information, a time series is analyzed at the highest possible frequency using different time scales. This tool differentiates between seasonality, detects structural interruptions and volatile groups, identifies a process's internal and external dynamic properties at different time scales, and studies unstable events in series. Wavelet filtering provides a natural way to deal with the different time characteristics found in series, rejecting the static assumption. However, this research will use wavelet analysis methods to analyze the time series of returns and market stock volatility based on scale.

Different time horizons change the structures between variables and, consequently, the decision structure from fixed to dynamic. High stock market returns increase confidence in price trends and the transfer of economic activities from unproductive to productive (production), thus increasing physical and human investment. So, the main purpose of this study is to analyze the effects of returns and stock market volatility on the economic activity of investors in Iran via different time scales using wavelet analysis, which is done for the first time in Iran. This new approach is based on the wavelet multi-scale method in which a time series is decomposed into different scales. The most important advantage of wavelet analysis is its ability to decompose data to multiple time scales. Many investors in the securities market do securities trading and make decisions on different time horizons; now, traders can be embodied as minute to minute, hour to hour, day to day, month to month, or year to year. In fact, due to the different time scales that different traders have for decision making, dynamic and realistic structure, effective nature of returns, and stock market volatility on economic activity, the relationship between different time scales can be changed. Economists and financial analysts have long considered the idea of analysis in decision-making for several periods. However, due to the lack of analysis tools, this fact has been limited to analyzing data over short- and long-term time scales. This study will provide an orthogonal wavelet analysis of covariance, correlation, and cross-correlation between stock market returns and economic activity at different time scales. Furthermore, variance and covariance of the different time scales will be analyzed to understand the true relationship between stock market returns and economic activity.

Osu et al. (2020) examined the common movements of the Nigerian stock market with the 11 N markets (Bangladesh, Egypt, Indonesia, Iran, Mexico, and Nigeria). They also examined the effect of volatility on market dynamics and used some wavelet-based measures to analyze market dynamics. The results showed a powerful joint movement among N11 countries, and noise had almost the same effect in terms of time-frequency for all N11 countries. It can be supposed that most of the market dynamics of N11 countries at lower scales (high frequency) are due to sharp volatility, while market principles drive higher scale dynamics (low frequency). Deviation and kurtosis can also determine the energy distribution in wavelet decomposition. Zhou et al. (2018)

studied the international stock market contagion: A CEEMDAN wavelet analysis. In this study, they used the CEEMDAN wavelet model (complete experimental mode analysis of the group with consistent noise) and examined the effect of contagion in stock markets (Asia, Europe, and the US) under different time frequencies. The results revealed that shocks caused by irregular events and severe accidents can be transmitted between different stock markets. Furthermore, shocks from irregular events can pose a sudden, short-term risk to stock returns. And shocks from severe accidents can pose a positive and lasting risk to stock returns. Lin et al. (2018) used wavelet analysis to examine the relationship between stock returns and bonds and stock market uncertainty. The results showed that the short- and long-term relationship between stocks and bonds showed many differences in different periods. It was also found that the relationship between bonds and stocks has a negative sensitivity to stock market volatility. It was also found that financial crises significantly negatively impact the relationship between bonds and stocks in the long run. Tiwari et al. (2017) used wavelet analysis to examine the relationship between inflation and stock returns over a long period (1790 to 2017) and at different frequencies. The results are compared with those in the United States and two developing countries (India and South Africa). Generally, the results of this study indicated that while the relationship between inflation and stock returns varies extensively at different frequencies and time intervals, there is no evidence that stock returns are a cover for inflation. Such a conclusion was true in both developed countries, the United Kingdom and the United States, and the two developing countries of India and South Africa. Boubaker and Raza (2017) conducted a study entitled "A wavelet analysis of mean and volatility spillovers between oil and BRICS stock markets". The results of this study reveal that oil prices and stock market prices are directly affected by the news and volatility of their market and are indirectly affected by volatility in prices other than wavelets. Likewise, volatility spillover effects and averages are affected by many overflows except in different time dimensions based on heterogeneous investors and market participants. Yilmaz and Unal (2016) used wavelet analysis to examine the volatility of Asian stock markets based on the FTSE100 and S&P 500 financial indicators. The results of wavelet analysis indicated that there are internal relationships between these stock markets, which have changed at different time intervals and frequencies. At the same time, it was observed that the stock markets of countries with advanced economies have greatly impacted the stock markets of Asian countries. However, the degree of dependence of Asian stock markets on strong global markets varies from country to country, and these differences can be seen in different periods. Çelik and Baydan (2015) show that stock exchanges with a high concentration of foreign investors have been highly affected by recent global financial crises. Furthermore, the discussion of asymmetric transmission using significant and different wavelet corollary results has been almost confirmed for some emerging economies. Masih and Majid (2013) conducted a study entitled Selectivity of Selected International Stock Market Indicators: A Continuous Wavelet Analysis and Cross-sectional Wavelet Analysis. The results of this study can be an important tool in decision-making for different types of investors. Kazemzadeh et al. (2020), using a discrete wavelet converter with maximum overlap and threshold autoregressive pattern of interaction of inflation and government budget deficit, studied from new angles two cases of total deficit and operational Deficit in the economy of Iran in 1990:1-2017:3. The results based on the wavelet converter show that in the horizons of more than 8 years, there is a causal relationship between both types of the budget deficit and bilateral inflation. Based on the results of estimating the threshold autoregressive pattern, in seasonal inflation less than 6.28%, the total budget deficit increases sharply in the face of the inflation shock. Likewise, the operating budget deficit before and after the threshold positively responds to the inflation impulse. Explaining that at seasonal inflation rates above 2.54%, the reaction intensity of this variable will be higher. In other words, the Tanzi effect is always stronger than the non-Tanzi effect. Fallahi et al. (2019) decompose a new approach using the wavelet analysis method that inspects the returns of a particular investment strategy into multiple investment horizons. The results of their research show that in investment companies with medium and low-risk aversion, with increasing investment horizon, the amount of investment in low growth stocks and the amount of investment in value stocks has increased. In contrast, the weight of growth and value stocks were not significantly different in the stock market portfolio survey. Seifollahi (2017) conducted a study entitled “Negative Relationship between Credit Risk and Currency Risk with the Price Return of Bank Stocks in Iran (GARCH-M approach)”. Regarding the research findings in the Iranian banking system, there is a negative relationship between credit risk and foreign exchange risk with the stock price returns of banks listed on the Tehran Stock Exchange. This indicates the innovation of this research. There is also a positive relationship between credit risk and foreign exchange risk and the risk of price returns on shares of banks listed on the stock exchange and securities. Rostami et al. (2016) show a significant relationship between the returns of various industries in the Tehran Stock Exchange and the returns of oil, gold, dollar, and euro markets. A stronger relationship exists between independent and dependent variables at shorter intervals. Besides, based on the sum of beta coefficients of independent variables in different periods and industries, it is determined that the variables of oil, gold, dollar, and euro prices have the most power to explain the index of different industries, respectively. Abbasi et al. (2016) investigated the relationship between trading volume, stock returns, and volatility in returns at different scales in the Tehran Stock Exchange. The results obtained from this study in the period under study show the difference in relationships between variables at different time scales, as in some scales, the Granger causality test confirms the existence of a causal relationship between time series.

3. Research methodology

This research is applied in terms of purpose, quantitative approach in data nature, and descriptive-correlational in data collection method. The statistical population of this research includes active indices in the Tehran Stock Exchange market; among all indices, the total index and price index have been used as samples. To this end, information about the stock market price index and the total index will be collected daily in the period 2011-2023; then, using wavelet analysis, time series are broken into different time intervals, and then with the correlation analysis method, the relationship and the effects of returns and volatility of stock market returns on the levels of economic activity in different periods will be studied. This research uses MODWT (maximal overlap discrete wavelet transform) to study the relationships between variables in Iran. The correlations between stock price indices and industrial production will be estimated at different time points of wavelet conversion.

The wavelet base comprises a parent wavelet representing the data's main process. Wavelet transform determines the number of frequencies in the signal and when those frequencies occur from the signal. Wavelet transform attains this ability by working at different scales. In wavelet conversion, we first consider the signal with a large scale or window and analyze its large features. In the next step, we look at the signal with small windows and get the small characteristics of the signal. In general, it can be said that wavelet transform acts as a compromise. On scales where time-dependent properties are more attractive, the wavelet transform has a higher resolution in the time domain, and on scales where frequency-dependent properties are more attractive, it has a higher resolution in the frequency domain. This type of compromise is exactly what the goal is in signal processing.

The discrete wavelet transform is implemented as a filter bank, acting as a sequence of low-pass and high-pass filters. We start with small scales corresponding to high frequencies to apply a discrete wavelet transform to a signal. Consequently, we first analyze the high frequencies. In the second step, we increase the scale by a factor of two (we decrease the frequency by a factor of two), in which case we analyze the behavior around half the maximum frequency. In the third step, we consider the scale factor 4 and analyze the frequency behavior around the quarter of the maximum frequency. This process continues in the same way to reach the maximum level of decomposition. Discrete wavelet transform returns two sets of coefficients as output: Approximation and Detail Coefficients. Approximation coefficients represent the output of the low-pass filter (Averaging Filter) in the discrete wavelet transform; the coefficients of detail indicate the output of the low-pass filter (derivative filter) in the discrete wavelet transform by applying the discrete Fourier transform again on the previous wavelet conversion approximation coefficients, we obtain the next wavelet transform, in each step, the sampling of the primary signal with factor two is reduced.

In each successive stage of the discrete wavelet transform, the approximation coefficients are divided into low-pass and high-pass parts. The wavelet transform is applied again on the low-pass part in the next step. As can be seen, our main signal is now converted into several signals, each corresponding to different frequency bands. Approximation and fractional coefficients under different bands are used in applications such as removing high-frequency noise from signals, compressing signals, or classifying different signals.

The idea of signal analysis with different scales is called multiresolution or multiscale analysis, and signal analysis in this way is also called sub-band coding.

In fact, in wavelet transform, like Fourier transform, a time-based function is expressed as a set of sentences with base wavelet functions, except that the wavelet function is not like sin and cos and contains a scale parameter.

Considering that the approximation coefficients include low frequency information and the general trend of the time series, it is necessary to select the layer that is most similar to the original signal from among the layers of approximation coefficients, and based on that, the optimal time horizon for holding shares to investment should be suggested.

The SSIM criterion (Structural Similarity Index Matlab) calculates the similarity coefficient, which examines the structural similarity based on the intensity of fluctuations and noises. In fact, with this criterion, we calculate the similarity between the original signal and the approximation coefficients to obtain the optimal time horizon of fluctuations. If the degree of similarity is high, this criterion approaches 1; if the similarity is very low, That approaches zero. The higher noise causes a decrease in this criterion.

$$ssim(x,\ y) = \frac{(2\mu x\ \mu y + c1)(2\sigma xy + c2)}{(\mu^2 x + \mu^2 y + c1)(\sigma^2 x + \sigma^2 y + c2)}$$

- μx:Average of approximation coefficient
- μy: The average of the studied time series
- $\sigma^2 x$: The variance of the approximation coefficient
- $\sigma^2 y$: Variance of the studied time series
- σxy: The covariance between the approximation coefficient and the studied time series
- *C1* و *C2* : constants for stability. (Wang et al., 2021)

4. Research results

The results of the first hypothesis test:

4.1 Wavelet variance analysis of stock returns

To analyze the data related to the variance of stock returns using wavelets, first, the data related to the total index are decomposed into 12 layers of approximation coefficients and 12 layers of detail coefficients. The number of layers is determined according to the complexity of the time series. The more complex the time series is, the more layers are needed to describe the data; therefore, the analysis of layers proceeds until the coefficients of approximation related to the last layer become zero and the coefficient of detail of the last layer becomes a simple wavelet pattern. The symbol denotes the approximation coefficients and the detail coefficients are denoted by the symbol d. The algorithm of decomposition of the main function into coefficients of approximation and coefficients of detail is such that in the first layer, the main function is decomposed into a1 and d1 and in the second layer, a1 is decomposed into a2 and d2 and this signal decomposition continues until the twelfth layer. The Diagrams For The Coefficients Of These 12 Layers Are Shown In The Figure (1):

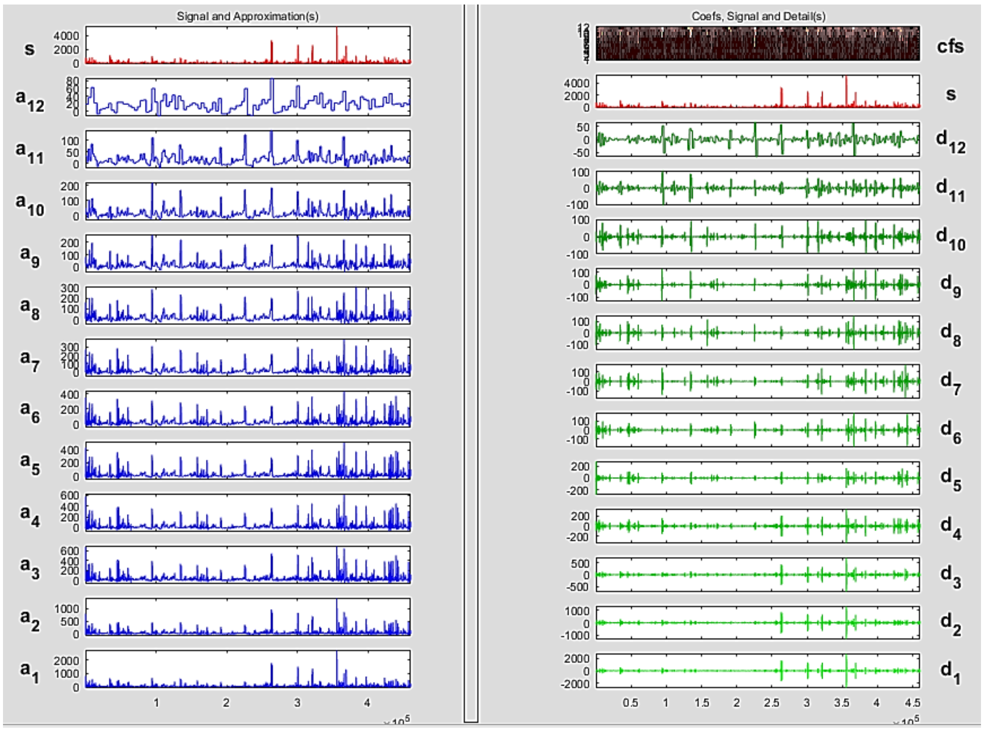

Figure 1. Diagrams related to approximation coefficients and detail coefficients in 12 layers

According to the diagrams above, the initial layers represent the signal's high frequency details, and the final layers describe the signal's low frequencies. The signal is displayed longer as we move from the initial to the final layers. In general, approximation coefficients show the trend of fluctuations longer than detail coefficients. The fastest dynamic corresponds to d1 and the slowest dynamic corresponds to the last layer. The detail coefficients examine the signal in a very short time. As seen from the above Figure (1), long-term regressions are approximated by layers of coefficients of approximation and short-term regressions are approximated by layers of detail. Table (1) shows the standard deviation and wavelet variance of each layer :

Table 1. Standard deviation and comovement variance of the industries at different time scales

Layer number	The standard deviation of approximation coefficients	Variance of approximation coefficients	Standard Deviation of Detail Coefficients	Variance of Detail Coefficients
1	57.607	3318.780	39.409	1553.190
2	50.358	2536.990	27.975	782.694
3	46.620	2173.590	19.037	362.467
4	44.877	2014.001	12.630	159.556
5	43.921	1929.155	9.209	84.839
6	43.030	1851.690	8.802	77.498
7	41.466	1719.590	11.491	132.069
8	39.394	1552	12.945	167.608
9	36.096	1303.100	15.775	248.909
10	31.058	694.540	18.360	337.140
11	23.850	568.870	19.905	396.267
12	17.058	291.015	16.668	277.861

Based on Table (1), the coefficients of approximation and detailed coefficients of variance of the stock returns volatility are given. As you can see, the coefficients of approximation of stock return volatility at smaller scales are lower, but with the increasing scale, the variance is reduced. This means that volatility about the mean of the short-term to long-term stock returns is lower. However, with the increasing scale, variance declined in the medium term, and in the long run, this deviation slightly increased and then decreased.

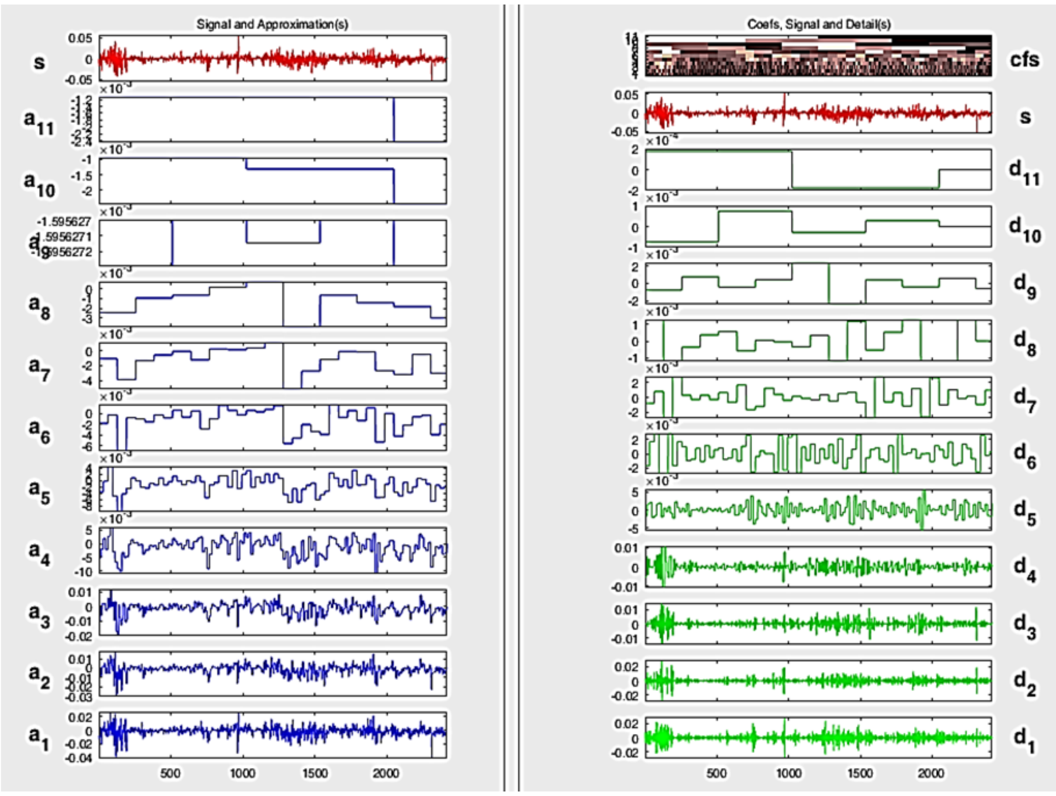

Figure 2. Diagrams of approximation coefficients and detail coefficients in 11 layers

4.2 Wavelet Variance Analysis Of the moving average of stock returns

The relevant data are decomposed into 11 layers of approximation coefficients and 11 layers of detail coefficients to analyze data on the Wavelet Variance Analysis of the moving average of stock returns. The first is exploited to extract long-term behavioral patterns, and the second is used to extract short-term behavioral patterns.

The diagrams for the coefficients of these 11 layers are shown in the Figure (2):

According to the diagram above, the initial layers represent the details of the signal's high frequency, and the final layers describe the low frequency of the signal. The signal is displayed longer as we move from the initial to the final layers. In general, approximation coefficients show a trend of fluctuations longer than detail coefficients. In fact, the fastest dynamic corresponds to d1 and the slowest dynamic corresponds to the last layer. The detail coefficients examine the signal in a very short time. As the above figures illustrate, the coefficient layers approximate long-term regressions and short-term regressions are approximated by the coefficient layers. Table (2) shows the standard deviation and variance of each layer:

Table 2. Standard deviation and comovement variance of the moving average of stock returns at different time scales

Layer number	The standard deviation of approximation coefficients	Variance of approximation coefficients	Standard Deviation of Detail Coefficients	Variance of Detail Coefficients
1	53.257	2837.363	28.673	822.197
2	48.595	2362.436	19.117	365.496
3	43.147	1862.516	15.361	235.990
4	39.354	1549.514	11.267	126.966
5	37.215	1385.690	8.484	71.994
6	34.972	1223.730	7.420	55.070
7	30.321	919.959	12.363	152.867
8	25.403	645.811	16.991	288.727
9	23.944	573.784	19.101	364.885
10	17.537	307.887	23.511	552.813
11	15.672	245.915	27.700	767.344
12	12.763	163.139	29.102	846.983

Based on Table (2), the coefficients of approximation and variance coefficients of the details of volatility and changes in the moving average of stock returns are given. The variance was used in the logarithm of variance for a better appraisal. As can be seen, stock returns moving average volatility approximation coefficients have been increased at fewer scales, but with the increasing scale, the variance is reduced. This means that the volatility of the short-term moving average in stock returns over the long-term average is less. Since variance in stock returns moving average at fewer scales (short-term) increased with increasing scale, the variance is much reduced in the medium term, and in the long run, this deviation increases again.

Table 3. Comparison of standard deviation and variance of stock returns wavelet wave and moving average stock returns

Stock returns	69.798	Standard deviation
	4871.960	Variance
Moving average stock returns	80.000	Standard deviation
	6400.158	Variance

As seen from Table (3), given that the amount of variance in stock returns is greater than the moving average of stock returns, then the fifth hypothesis of the research is accepted. Likewise, based on the moving scales of each stock return and the moving average of stock returns in Tables (1) and (2), they had less wavelet variance and less comovement during long-term scales, but over short time scales, the movement increased and the variance of the wavelet efficiency was higher among them.

4.2.1 The similarity coefficient between the original signal and the wavelets of different time layers

Table 4. The similarity coefficient between the original signal and the wavelets of different time layers

	The highest SSIM similarity index value among approximation coefficients	Layer number of optimal approximation coefficient	Optimal time horizon
Volatility of stock returns	0.961	A5	32-64
The volatility of the moving average of stock returns	0.970	A6	64-128

According to the results of Table (4), the approximation coefficients related to the fifth and sixth layers have been able to have a higher similarity and movement with the fluctuations of stock returns and the fluctuations of the moving average of stock returns, and among other time levels, they are more optimal time horizons to hold Stocks and invested.

Given the first hypothesis of the research, which is about the higher frequency coefficient and fluctuations of stock returns compared to the moving average of stock returns, because the amount of variance of stock returns is more than the moving average of stock returns, therefore the first hypothesis of the research is accepted; Also, according to the fluctuations in each of the scales of stock returns and the moving average of stock returns, during the long-term scales, the wavelet variance has been less and less comovement.

4.3. Test results of the second hypothesis

4.3.1 Wavelet Analysis of the moving average of the total index

The data related to the moving average of the total index is analyzed using wavelets. First, the relevant data are decomposed into 12 layers of approximation coefficients and 12 layers of detail coefficients. Long-term behavioral patterns are extracted using diagrams and coefficients of approximation layers, and short-term behavioral patterns are extracted using diagrams and layers of detail coefficients. Now, these data are analyzed using the wavelet.

The graphs related to the coefficients of these 11 layers are shown in the Figure (3):

According to the diagram above, the initial layers represent the details of the signal's high frequency, and the final layers describe the low frequency of the signal. The signal is displayed longer as we move from the initial to the final layers. In general, approximation coefficients show a trend of fluctuations longer than detail coefficients. The fastest dynamic corresponds to d1 and the slowest dynamic corresponds to the last layer. The detail coefficients examine the signal in a very short time. As can be seen from the above figures, long-term regressions are approximated by coefficient layers and short-term regressions are approximated by detail layers. Table (5) shows the standard deviation and variance of each layer:

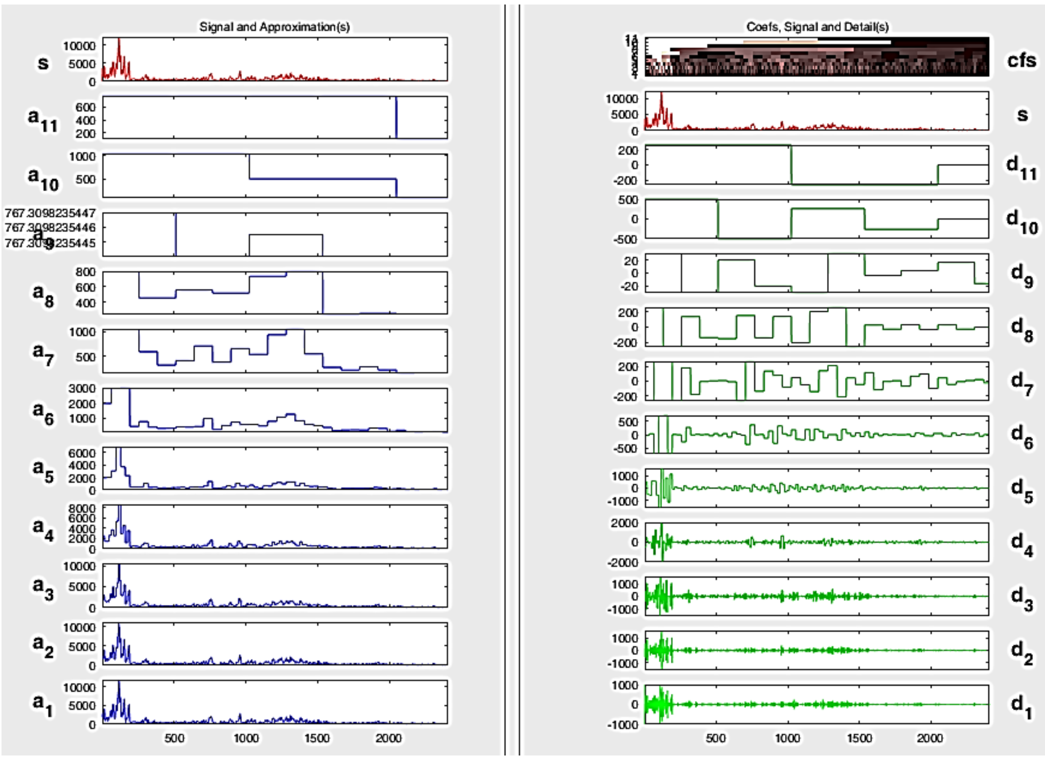

Figure 3. Diagrams of approximation coefficients and detail coefficients in 111 layers

Table 5. Standard deviation and comovement variance of the moving average of the total index at different time scales

Layer number	The standard deviation of approximation coefficients	Variance of approximation coefficients	Standard Deviation of Detail Coefficients	Variance of Detail Coefficients
1	53.743	2888.417	36.371	1322.921
2	50. 963	2597.328	30.404	924.463
3	47.839	2288.665	28.173	793.773
4	45.101	2034.189	22.711	515.834
5	40.326	1626.266	18.202	331.348
6	38.540	1485.408	15.347	235.560
7	31.503	992.501	10.844	117.613
8	25.680	659.513	9.214	84.917
9	22.442	503.687	7.302	53.332
10	17.529	307.300	6.284	39.595
11	12.767	163.021	5.928	35.150
12	8.783	77.158	4.315	18.626

Based on the table above, the wavelet variance and investment risk are reduced as we move from the initial to the middle layers. According to Figure (3) and the results of Table (4), the variance of the coefficients of approximation and the coefficients of the details of the volatility and changes of the moving average of the total index are given. It was used for a better review and to observe the variance in the logarithm of variance. As can be seen, the coefficients of approximation of the volatility of the moving average of the total index lower scale increases, but the variance is reduced with increasing scale.

Table 6. Comparison of standard deviation and variance of total index wavelet returns and moving average of the total index

Standard deviation	102.120	Overall Index
Variance	10428.425	Variance
Standard deviation	1093.80	Moving average of the total index
Variance	1196400	Variance

As it is clear from Table (6), since the amount of variance of the moving average of the total index is more than the total index, the second hypothesis of the research is rejected. Likewise, based on the movement scales of each of the total indexes and the moving average of the total index in Tables (4) and (5), they had less wavelet variance and less movement during long-term scales. Still, both comovement and wavelet yield variance were more common during short-term time scales.

4.3.2 The similarity coefficient between the original signal and the wavelets of different time layers

Table 7. The similarity coefficient between the original signal and the wavelets of different time layers

	The highest SSIM similarity index value among approximation coefficients	**Layer number of optimal approximation coefficient**	**Optimal time horizon**
Fluctuations of the total index	0.964	A6	64-128
Fluctuations of the moving average of the total index	0.921	A8	256-512

According to the results of Table (7), the approximation coefficients related to the sixth and eighth layers have been able to have a higher similarity and comovement with the fluctuations of the total index and the fluctuations of the moving average of the total index, and among other time levels, they are more optimal time horizons for maintenance Stocks and investment.

The second hypothesis of the research is about the higher frequency coefficient and fluctuations of the total index compared to the moving average of the total index; because the amount of variance of the moving average of the total index is more than the total index, the second hypothesis of the research is rejected. Also, according to the movement scales of each of the total index and the moving average of the total index, during the long-term scales, the wavelet variance was less and the movement was less, but during the short-term time scales, the movement increased and the wavelet variance of the return among them has been more; Therefore, considering that the amount of fluctuations and the frequency of the moving average of the total index is much higher than the total index, the second hypothesis of the research is rejected.

5. Conclusion and discussion

According to the first hypothesis of the research, which is about the higher frequency coefficient and volatility of stock returns than the moving average of stock returns, since the variance of stock returns is more than the moving average of stock returns, the first hypothesis of the research is accepted. Similarly, regarding the moving scales, each of the stock returns and the moving average of stock returns during the long-term scales had less wavelet variance and less comovement. Still, over short time scales, the movement increased, and the variance of the wavelet return was higher between them. According to the second hypothesis of the research, the frequency coefficient and volatility of the total index are higher than the moving average of the total index, considering that the amount of variance of the moving average of the total index is more than the total index, so the second hypothesis of the research is rejected. Likewise, according to the movement scales of each of the total indices and the moving average of the total index during long-term scales, the wavelet variance was less and had less comovement. Still, over short time scales, there was more comovement and the variance of the wavelet efficiency was higher between them.

Based on the analytical results of this research, traders are suggested to pay more attention to analytical methods based on moving averages to analyze and develop their trading strategies in the short term and give more points to the price and yield of the shares in the long term to analyze their trading methods and formulate their strategies based on the stock price and return. Also, due to the higher wavelet variance of the return in the short term and its lower in the long term, non-professional traders can choose short- and long-term investments based on their strategy. Given the comparison of the wavelet variances of the stock return and the moving average of stock returns in different time scales and based on the degree of similarity of each of these two variables with the wavelet variance of the approximation coefficients, the fifth and sixth layers are the optimal layers for making decisions in Fluctuations were selected. Based on the obtained results and considering that the wavelet variance of the stock return is lower than the variance of the moving average of the stock return, it is suggested that traders should use methods based on the price itself, such as price action methods, in their forecasts. Also, based on the SSIM similarity criterion between the main signal with the approximation coefficients related to stock returns and the moving average of stock returns, it is suggested that traders who aim for long-term investment should consider the fifth and sixth levels of the time horizon to hold stocks. For traders who want to act in the short term, give more weight to price-based analysis methods to determine their trading strategies and use technical analysis methods to identify their entry and exit points. Medium-term shareholders can also use a combination of price-based and moving average methods (by assigning more weight to price-based methods). Based on the results of the layer analysis, it seems that long-term investment is not justified. However, the strategy of real people is different from that of investment institutions on different time horizons. However, the logical solution seems to be to sell shares after the end of an upward trend and temporarily exit the market during the recession, a period of correction and extreme erosion. In general, by comparing the variances of the total index and the moving average of the total index and examining the similarity indices between the main indices and the approximation coefficient, the sixth time layer for the total index and the eighth time layer for the moving average of the total index were selected as optimal time horizons have been and the characteristics can justify this reason for the difference in the optimal time layers compared to the original price. According to the lower wavelet variance of the total index compared to the moving average of the total index, it is suggested that analysts and traders allocate a greater share to the fluctuations of the total index in their analyses and make decisions based on price-based analytical methods such as price action and Fibonacci. Also, they can invest in index-making stocks in the optimal period (sixth-time level). Also, based on the research results, it is suggested that beginner traders adjust their strategies based on the optimal time layers obtained for each of the analyzed variables, which were usually medium-term, and professional traders should choose A combination of short-term and long-term investments.

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